

1970

# Digital optimization of long-range power system planning using the dynamic programming technique

Talaat Ahmed El-Tablawy  
*Iowa State University*

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>

 Part of the [Electrical and Electronics Commons](#), and the [Oil, Gas, and Energy Commons](#)

## Recommended Citation

El-Tablawy, Talaat Ahmed, "Digital optimization of long-range power system planning using the dynamic programming technique " (1970). *Retrospective Theses and Dissertations*. 4301.  
<https://lib.dr.iastate.edu/rtd/4301>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact [digirep@iastate.edu](mailto:digirep@iastate.edu).

71-7262

EL-TABLAWY, Talaat Ahmed, 1935-  
DIGITAL OPTIMIZATION OF LONG-RANGE POWER  
SYSTEM PLANNING USING THE DYNAMIC PROGRAMMING  
TECHNIQUE.

Iowa State University, Ph.D., 1970  
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan

DIGITAL OPTIMIZATION OF  
LONG-RANGE POWER SYSTEM PLANNING  
USING THE DYNAMIC PROGRAMMING TECHNIQUE

by

Talaat Ahmed El-Tablawy

A Dissertation Submitted to the  
Graduate Faculty in Partial Fulfillment of  
The Requirements for the Degree of  
DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University  
of Science and Technology  
Ames, Iowa

1970

## TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. REVIEW OF LITERATURE	6
III. DATA REDUCTION PROGRAM	22
IV. LOAD MODELING PROGRAMS	25
V. CAPACITY MODELING PROGRAM	29
VI. INFLUENCE OF GENERATOR SIZE ON THE RELIABILITY AND THE COST OF POWER SYSTEM	33
VII. COST OF EXPANSION PATTERNS	37
VIII. OPTIMIZATION OF EXPANSION PATTERNS' COSTS USING THE DYNAMIC PROGRAMMING TECHNIQUE	49
IX. CONCLUSION	61
X. LITERATURE CITED	63
XI. ACKNOWLEDGMENTS	68
XII. APPENDIX A. DATA REDUCTION PROGRAM	69
XIII. APPENDIX B. LOAD MODELING PROGRAM	80
XIV. APPENDIX C. LOAD FORECASTING	104
XV. APPENDIX D. LOAD DURATION CURVES	138
XVI. APPENDIX E. SYSTEM RELIABILITY MEASUREMENTS	159b
XVII. APPENDIX F. EFFECT OF NEW UNIT SIZE ON RELIABILITY AND SPINNING RESERVE OF THE SYSTEM	243
XVIII. APPENDIX G. COST PROGRAM	269

## I. INTRODUCTION

Several digital expansion programs have been written in the past few years. They have been used to plan and evaluate the investment costs for a single area. Usually, several alternative patterns of new units added to the system are chosen and the loss of load probabilities are computed for each expansion plan to assure that the plans are comparable in their reliability. The total costs of each expansion pattern is evaluated and the cheapest pattern is selected as the best one. However, this technique provides no assurance that the best pattern of those considered is indeed the optimum pattern. This is because the expansion patterns studied by this method are not necessarily the only ones which meet the predetermined reliability index. There is still the possibility of having other patterns which could be more economical.

In this research, a new method of selecting the optimum pattern is introduced. Here we divide the interval of time for future planning into a set of subintervals two to three years long. Considering the first subinterval of time alone, we try to find the alternatives of new unit additions which will satisfy the selected reliability index and compute the cost of each alternative pattern. Then proceeding to the second subinterval, we again find an equal number of alternatives that will satisfy the reliability requirement, and

compute the cost for these alternatives as before. The process is continued to the end of the planning period. If the number of the subintervals is equal to  $N$  and the number of choices or alternatives is  $r$ , we will end with  $r^{N-1}$  possible decision sequences (1). For example, if  $r = 4$  and  $N = 6$ , then we would have  $4^5$  or 1,024 possible combinations to examine. Since it is not feasible to study this many possibilities in a reasonable time, we seek a better method of optimization. One possibility is to use the dynamic programming technique which was introduced by Bellman (2) and which has gained considerable popularity in recent years because of its applicability to digital computers. The essential feature of dynamic programming is that it reduces the  $N$ -stage decision process to a sequence of  $N$  single-stage decision process which makes it easy to solve the problem described above. This reduction is made possible by use of the fundamental principle of optimality (3) which states that:

An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

The number of comparisons to be made are reduced now to  $2r + (N-2)r^2$  or equal to 72 compared to 1,024 in the previous case. This method may also be expanded to include not only the unit size but also the choice of the location for each new added unit. This constitutes a second dimension to the problem.

The dynamic programming technique will be described in this thesis. Preparatory to this problem, however, are four major computer programs which are necessary preliminary steps.

These programs are:

1. The data reduction program which reads and reduces to common format all hourly load data from all six Iowa Pool participants.
2. The load modeling program which constructs detailed load models from the historical data, including load forecasting and load duration curves.
3. A probability program which evaluates the reliability index for the generating units, computes the amount of the spinning reserve<sup>1</sup> for the system and determines when the new generating capacity is required.
4. An investment and production costing program which may be used to evaluate the cost of the new additions as well as the operating and maintenance cost for all units in the system.

These four programs are described in the four chapters which follow. The first chapter is entitled "Data Reduction Program". This program reads any individual company data set. After then it converts this data set to a common format and

---

<sup>1</sup>Spinning reserve is the amount of capacity available in excess of peak load to provide for forced outages.

stores it. The process is repeated for the other five companies and the six data sets are merged to form one data set called the "Pool Data Set".

The second chapter is entitled "Load Modeling Programs". Two programs are written to construct the load model. The first program is called "SUBROUTINE TREND" which examines the statistical nature of the "Pool Data Set" as well as the individual company sets. This subroutine finds out the statistical parameters for the data set every month and samples the observations required to predict future loads. These observations are statistically tested and the loads are predicted for the future using an exponential load model (4). The second program is called "SUBROUTINE LOADUR" which constructs the load duration curves for each month of the year. These curves are required in evaluating the production costs.

The third chapter is entitled "Capacity Modeling Program". This program computes the reliability index using the loss of load probability (LOLP) and the loss of capacity (LOCP) methods. It also computes the spinning reserve necessary to supply the load at the predetermined reliability index. The program also tests the effect of the size of the new unit or units and recalculates the spinning reserve for the new case.

The fourth chapter is entitled "Cost of Expansion Patterns". Here we compute the costs of the investment for generation, the investment of interconnection capacity, the investment of the new transmission additions, the operation



and maintenance costs, and the cost of fuel. The present worth value of the total costs is computed and stored for further application in selecting the optimum pattern.

These programs will be discussed in greater detail later.

## II. REVIEW OF LITERATURE

Power system planning is not a new branch of science. In the early days, the planner examined the past load data and predicted the amount of new capacity required to supply the estimated load. The next step was to select the size of the generator or generators to be added to the system from the list provided by the manufacturers without particular regard to the reliability consideration. In doing so, he ran the risk of service interruptions as well as insufficient capacity due to errors in his predictions. Owing to the limitations of confidence in such predictions and the rapid changes of the generator characteristics, he was not able to plan the generation for a long period of time.

In July 1899, H. M. Atkinson (5), gave a paper in Atlanta about generation planning where he said,

With a successful record in the past, with everything running smoothly and satisfactorily, and with a bright outlook for a constantly increasing business, it is a hard thing to face radical and expensive changes, but the growth of our business forces us to act. We concluded early in the spring that we must have increased capacity to carry our load during the coming fall and winter. Two alternatives faces us: First, should we go on and add in conformity to our present equipment, or, second, should we depart radically from our past methods and, as it were, take a fresh start?

He also pointed out that since the manufacturers had chosen sixty hertz as their standard (it was 125 and 133 hertz before) a change in frequency was also involved.

He discussed the choice of the primary voltage and then the choice of the generator and then he added,

Next we come to the phase of the generator. This has been, perhaps, the hardest question to solve satisfactorily. There is not, perhaps, so much in the question of whether three-phase or two-phase is best. It seems best to us to employ the form of generator that is most economical in copper cost for motor distribution.

After further discussion, he decided to select the three-phase generator of 400KW capacity and rated 1,000 volts and promised that any new generator would be of larger size such as 1,000KW.

In the period that followed, many planners used the same approach to select their next new additions.

In 1930, R. Bailey (6) wrote a paper which offered a new approach to the planning problem. Plotting the yearly peak loads and the net guaranteed KW-HR since 1902, he was able to predict that the load was to double every five years. He discussed the inherent advantages of any system planned that there was sufficient generating capacity at various large load centers to carry the load under emergency conditions. The selections of the new units were made according to that principle and the arrangement of the power system was to permit sufficient transfer of power throughout the system so that the generating stations could not only be loaded so as to secure the maximum economy, but also so that minimum generating capacity be provided for emergency use.

In 1931, R. C. Powell (7) wrote a second paper on power system planning for the Pacific Gas and Electric Company. In that paper, he predicted the future loads not only for the

immediate needs as before, but also for at least fifteen years. A comprehensive plan was developed to provide power at the lowest possible cost. Several plans were developed taking into consideration sizes of units, dates of installations, obsolescence, operating reliability and flexibility and the total cost of each plan was computed including the investment costs, maintenance and operation costs, fuel costs, and the transmission costs.

The plan with the lowest cost was adopted to be the generation expansion plan.

In 1932, A. P. Fugill (8) of the Detroit Edison Company published a paper in which he defined an emergency criterion to determine the time and the size of the new generation capacity additions. The criterion for station capacity was based on the peak load with the condition that it must be possible to carry the predicted load during a reasonable emergency. The probable total system load peaks were predicted for several years in advance from a study of past performance, the existing trend and other factors which affected that load growth. The probable loads on individual substations were predicted from the same data. From a knowledge of the probable total system load, the probable individual substation loads, the proposed changes in load areas and the diversity factor between the peak loads of those substations, the peak load on each power station was estimated.

To determine the firm capacity of the total system, the emergency criterion which had been established was to assume

that the peak load occurred when the largest unit was down for routine maintenance and the next largest unit was out of service due to an emergency. The new unit was added if the firm capacity was less than the sum of peak load plus predicted load variation. Several alternative generation expansion patterns were considered and the total costs were computed. The cheapest pattern was selected for future planning.

Different papers appeared in literature after that on the same topic with different emergency criteria (reliability measurement) assumed by each planner to meet the power system needs. The assumption of a particular emergency criterion affects both the choice of the new unit size and the overall costs considerably.

In 1933, W. J. Lyman (9) published a paper which was the first step in the application of probability theory to compute the loss of capacity probability. In that paper, he emphasized that the three most vital problems around which the whole fabric of future planning was woven are long-range load forecasting, the relation between load and capacity, and fixed capital replacements. He pointed out that the long-range forecasting requires more than anything else, an understanding of the law of inevitable changes and the ability to project into the future and detect basic trends and new influences which are obscure or invisible in the present. He also discussed the relation between spare (reserve) capacity and service continuity (reliability) and defined a measure of the

reliability of the system in terms of the number of outages per year and the total duration of such outages. The conclusion was that the addition of a greater number of units to an interconnected system reduced the percentage of loss of capacity resulting from any given outage, but it also increased the probability that such an outage will occur.

In 1934, S. A. Smith Jr. (10) wrote a fine paper to compute the spare capacity by probability theory using the binomial expansion method. He pointed out that the spare problem resolved itself into two distinct parts, namely service reliability and the expectation of load outage. At the end of that paper, he discussed the effect of extra units added to the system with regard to both economy and reliability. In another publication (11) he calculated the reliability of the system in terms of outage duration and magnitude of that outage.

In 1947, W. J. Lyman (12) published a paper on the application of the probability theory to compute the capacity outages. He presented a practical method for evaluating outage probabilities of generating capacities by assuming a forced outage rate for each unit and he was able to combine the units in each generation station of the system under study using the binomial expansion method. The result for each station was plotted and then he combined all the curves to form a total combined curve that gave the outage probability of generating capacity. Using these results, he was able to determine the proper amount of reserve capacity and also to study the effect

of interconnections. The effect of the size of the new unit on the reliability of the system was also studied using the previous method.

In the same year, H. P. Seelye (13) wrote a paper on the probability calculations.

The frequency, duration, and interval between two successive outages were computed and he discussed the effect of a new addition on both the reliability and the spinning reserve of the power system.

After that, many papers appeared in the literature in the application of probability theory in evaluating the reliability and the spinning reserve. All methods used to measure the system reliability are classified to three different methods, namely the loss of load probability (LOLP), the loss of capacity probability (LOCP), and the loss of energy probability (LOEP).

In 1948, M. L. Waring (14) wrote a paper on system planning. From the history of load data, he predicted the future load. With the use of loss of load probability, he measured the system reliability (taking one day per eight years as a risk index). The spinning reserve was about 16 percent of 1952 peak load. Several unit sizes were then added to the system to satisfy the reliability needs and to keep the spinning reserve as low as possible and the overall costs were then computed. The expansion pattern that had the lowest cost was then selected for future planning.

In 1950, A. L. Williams and E. L. Kanouse (15) wrote a paper on power system planning in the city of Los Angeles. They undertook a survey to the electrical power demands of the city by load classes and by geographical areas. The method used was essentially a field survey of land use by the various classes of load, coupled with application of load demands which are related to unit square miles of the load class. The population of the city in the previous years was recorded and the trend of population increase was computed. Housing requirements for the excess in population were estimated. After that survey, the basic system plan was estimated for the city that covered 400 square miles at that time. That plan provided that the future load be distributed from not less than six receiving stations, each of a maximum capacity equal or less than the source of power feeding that station.

In 1955, L. K. Kirchmayer, A. G. Mellor, J. F. O'Mara and J. R. Stevenson (16) published a paper that recorded the method of analysis and the results obtained from a study made to determine the optimum economic size of steam-electric generating units that should be added to a certain power system. The factors such as size of system, size of units added, forced outage rates, rate of load growth, installed cost of larger generating units, the effect of maintenance programs, and the effect on the transmission system were discussed. They considered a system of 2,000MW and expanded that system to 10,000MW by four different patterns.



1. 8% to 5% expansion pattern.  
i.e., the size of all units was kept within a band of 8% to 5% of the total installed capacity of the system.
2. 10% to 7% expansion pattern.
3. 15% to 10% expansion pattern.
4. 250MW expansion pattern.

The reliability index chosen was 1 day in 11 years. The method of analysis used was to determine the most economical pattern of system expansion which consists of two parts:

1. The determination of the spinning reserve capacity required on the system to meet the assigned index of reliability.
2. The application of the assumed cost factors to each pattern of expansion to determine the most economical pattern of expansion.

They concluded that if the investment cost of large units continues to decrease with size and the forced outage rate for large units remains at its present level, the most economical pattern of system expansion was to add units of between 10% and 7% of the size of the system studied. Also, it was estimated that the unit sizes of the order of 500 to 600MW would be economically utilized on some power systems within the following 10 years:

In 1956, M. J. Steinberg and V. M. Cook (17) wrote a paper which described a method for the evaluation of steam-electric capacity additions to an expanding system and studied the relative economic merits of adding different unit sizes to the system of the Consolidated Edison Company of New York, Inc.

The reliability index was selected such that the reserve capacity was maintained equal to 15% of the annual peak load as a first alternative. The second one was to maintain the reserve capacity equal to the sum of the capacity of the largest units on the system. The expansion was computed for 50 years by adding similar unit sizes for the three expansion patterns chosen. The total costs of the three expansion patterns were computed and converted to its equivalent present worth values and the minimum cost pattern was selected as the optimum one.

In 1957, L. K. Kirchmayer, A. G. Mellor, and H. O. Simmons Jr. (18) published a paper on the effect of interconnections on economic generation expansion patterns. In a later paper, they extended the work done before (19) to include an evaluation of economic benefits of interconnections of areas with regard to a reduction of reserve capacity to cover outages and the optimum unit size expansion patterns for integrated areas as a function of initial system size and interconnection distance. Three different system sizes were selected and the expansion of each system was carried out for a 30-year period. They concluded that interconnection of areas results in reduction of installed capacity because of load diversity between the areas and in economic interchange of power resulting from difference in incremental production costs.

Also, the economical unit size percent expansion pattern becomes smaller as the system load increases.

They found, also, that the cost of transmission facilities

within an area may also reduce the economic unit size expansion pattern. The reliability index during all these studies was such that the probability of loss of load did not exceed one day in 10 years.

An application of such a study was done on the Dayton Power and Light Company using an IBM 650 computer using three different expansion patterns (19).

In the same year, with the help of digital computers, M. K. Brennan, C. D. Galloway, and L. K. Kirchmayer (20), published a fine paper on the loss of load probability computation using a digital computer (IBM 650). The system chosen had 25 units and the time for the computations was 5 minutes. A continuation of that paper was published by the last two authors (21) on the usage of the digital computer to evaluate the overall costs of expansion patterns. Those two papers opened the door to computerization of power system planning.

In 1959, C. J. Baldwin, D. P. Gaver, and C. H. Hoffman (22) wrote a paper on mathematical models for use in the simulation of power generation outages. In that paper, they tried to find the optimum plan to follow in expanding system generation and transmission to meet increasing loads. They used the operational gaming or system simulation to answer these questions. They stated that operational gaming employed a combination of system analog, Monte Carlo techniques, and simulated human decisions. The system analog is a mathematical and logical model of that system used to represent a sequence of

system events in the future. Random occurrences of events are obtained using the Monte Carlo technique. These events could be unit forced outages, deviation of daily load estimates, random variation of daily peak loads, and others. The logic of system operation which is the human element is built into the model. Actual system events are simulated for every day. Evaluation of the risk of losing load determines the dates of each generator addition. Then the economic evaluation of the particular pattern of expansion is computed. Comparing such patterns, the planning policy changes could be evaluated. The system model consists of two parts:

1. Load model in which forecasted load is computed.
2. Capacity model in which the reliability index is computed and checked against the design index.

The continuation of that paper (23) gave some numerical results using Public Service Electric and Gas Company (PSEG) system as a model.

J. K. Dillard and H. K. Sels (24) tried to determine the answer to the previous question using the same operational gaming method. They discussed the influence of new unit addition sizes on both reliability and spinning reserve. The system studied was Pennsylvania-New Jersey-Maryland Interconnection. The plan of attack to the problem was to select the alternative patterns and then compare all of the costs and select the lowest cost expansion pattern.

D. N. Reps and J. A. Rose (25) wrote a paper on game theory application to answer the same question. They considered

planning in the future is like a game in which management must commit itself beforehand for capital improvements of quality and degree of efficiency adequate for forecasted needs. They defined operational gaming as a means of investigating effect of chance events upon results of following a particular strategy or policy (26). It requires explicit numerical estimates of chance influences such as load fluctuation probabilities and generation-unit reliabilities. On the other hand, if reliable estimates of chance events are not available, the game theory, which endeavors to discover strategies or policies that are advantageous in the face of uncertainties, must take over. In that paper, three possible load growths were assumed and three expansion patterns were proposed. The total costs were computed and the least costly pattern was adopted.

Another paper (27) was published a few months later in which the authors explained how to apply the game theory in power system planning. They constructed gaming models such as system data, system operating, and planning rules and stored those models in a digital computer memory. The system behavior was simulated on a daily basis. They classified the gaming models used in planning into two different types of models: deterministic and stochastic (chance or random).

K. M. Dale, W. H. Ferguson, C. H. Hoffman, and J. A. Rose (28) published a paper on production cost calculation for system planning by operational gaming models. The method of solution adopted for generation planning problem is system simulation

by a mathematical model. The model is operated for a period of time and this is considered as one of the games. Then other games are played with other sets of statistically correct, but randomly chosen input data. After enough games are played, the results from a logical pattern are recorded. The production cost calculation method adopted is an improved version of the commonly used load-duration curve method in which the energy-producing units are stacked up according to their economical priority to fill the area under the curve. The spinning reserve and pumped-storage capacities are considered in the model.

In 1960, C. J. Baldwin, C. A. DeSalvo, and H. D. Limmer (29) published a paper on the effect of unit size, reliability and system service quality. Operational gaming or system simulation was used in their program. They concluded their study by pointing out that larger units are more economical than small ones. Better heat rates, reduction in operation and maintenance charges are some of the merits of large units. On the other hand, they require increased spinning reserve and may cause severe transmission cost penalties by undue concentration of generation.

In 1961, R. J. Fitzpatrick and J. W. Gallagher (30) published a paper of optimization of generator expansion pattern. A series of computer programs were developed using IBM 650. Two detailed programs showed how the annual load-duration curve was used to propagate a series of new generator-requirement curves. The general method of solving this problem starts with

constructing the annual load duration curve. All capacity and energy under that curve plus a fixed percentage reserve are assumed to be supplied entirely by the existing units. The new unit added will be required to operate within a load band determined by calculating the point on the load duration curve where the sum of annual fixed costs and annual operating costs for that unit equals the corresponding sum for the new generator with the next highest operating cost. At the end of that step, we will have several portions or areas under the load-duration curve. The capacity of the existing generation to supply each portion of that curve is then computed. The deficiency in generation for a particular portion will be the new generator size to be added to that area. Several units could be required to supply the load for that year. The process will be repeated for every year and a series of expansion patterns could be obtained by that method. Several patterns could be computed by changing the spinning reserve capacity.

In 1963, E. S. Bailey, Jr., C. D. Galloway, E. S. Hawkins and A. J. Wood published two papers (31), (32) on generation planning programs for interconnected systems. In the first paper, they described a system of digital computer programs which may be used to plan and evaluate the generation-interconnection system expansions of two areas. These programs include a load modeling program, a capacity modeling program and an investment costing program. These programs are based on theoretically sound analyses and permit the use of

reasonable approximations to achieve their objective. Using these programs, the interconnected systems may be expanded to meet growing loads and maintain adequate levels of reliability. The investment, economic and reserve capacity benefits may be determined. The loss-of-load probability of each area is computed and a new unit is added to the area that has the poorest reliability index.

In the second paper, the authors described a set of digital computer programs for evaluating the production costs for the two areas by simulating the hour-by-hour operation of the system and scheduling all units on an equal incremental cost basis. Several expansion patterns are proposed and the least costly pattern is selected.

In 1964, C. D. Galloway, L. L. Garver (33) published their paper on generation expansions for a single area. As in the previous two papers, a series of computer programs were developed and written in Fortran IV for use on the GE 625. The loss-of-load probability was used to determine the time of the new additions assuming that the preselected risk level is exceeded. The effective capability of the new unit was considered in estimating the spinning reserve. The size of the units were prerecorded and the program added the units according to their order. The overall costs were computed for alternative patterns and compared.

In 1966, L. L. Garver (34) measured the effective load carrying capability of a new generating unit. The measurement



is made at some designated level of reliability. He presented some graphical methods used for estimating that capability. In that paper, he also discussed the effect of unit size on both the effective capability and the spinning reserve. He showed that adding a second unit of the same size will result in an increase in the effective load capability of that unit over the preceding one. The expansion pattern could be selected such that the effective load capability of the ordered units will match the corresponding forecasted load growth at the design reliability index.

K. D. Dale (34-a) wrote a paper on the application of the dynamic programming method to the selection and timing of generation plant additions. A constant percentage reserve is assumed and the forecast peak-load trend line is obtained by extrapolating the curve through the actual annual peaks during the last ten years. The installed capacity is obtained by adding the reserve capacity to the forecast peak load. Two plans were adopted. The first plan makes use of a few large units installed at intervals of three years whereas the second plan advocates the addition of a smaller sized unit each year. He then applied the dynamic programming technique to select the optimum expansion pattern without measuring the reliability of the system or the effective capability of the new units added.

H. Balériaux (34-b) published a paper on the dynamic

optimization of a range of generating plant. He used the curve of the production envelope associated with the consumption curve to determine the most economic composition of a range of available generating plant. He divided the plant into three categories, peak load plant operating say 0 - 1,000 hours per annum, medium service plant operating 1,000 - 4,000 hours per annum, and base load plant operating 4,500 - 8,000 hours per annum. For each group, the generating cost may be assessed in terms of fixed and variable costs and it can be ascertained on this basis, what is the most economic combination in terms of the proportion of each class of generating plant.

In 1968, R. R. Bennett (34-c), wrote a paper in which he forecast the unit and plant sizes for the next 20 years. He predicted that our large station in the 1980's will include 3,000 MW units. The trend toward consolidation of electric systems, pooling of generation, and participation in joint generating stations will result in economic justification for increasingly larger stations. He also expected that four of these large units could be located at a favorable site for an aggregate capacity of 12,000 MW. He then added that the average unit size in the late 1980's will probably be about 1,500 MW and the average station size in service at that time will be 4,000 MW which may be located underground or under the sea, which would reduce property requirements.

In 1969, H. Ogawa (34-d), wrote another paper on the determination of service dates of the new units added to the power system. He proposed the use of the ratio of the mean to the standard deviation of the probability distribution of the supply margin, using the probability models of hydro and thermal power supply capabilities and system load demand, to measure the system reliability. The units are added if the reliability index measured is less than the predetermined index.

In the same year, Robert J. Ringlee and Allen J. Wood (34-e), published a paper on system reliability calculations. Instead of assuming fixed outage or load duration intervals, they used an exponential distribution of durations. The generation system model is based on a Markov chain analysis and assumes statistically independent, stationary, exponential distribution of available and repair times for each machine.

Charles D. Galloway, Len L. Garver, Robert J. Ringlee, and Allen J. Wood (34-f) continued the previous work and they published a paper on generation system planning technique employing Markov chain representations of generation system and load models as before. They studied a medium size system applying this technique.

In 1970, R. F. Karliceck (35) published a paper on evaluating power system expansion plans. A digital program to assist engineers in evaluating these plans was described though no mathematical details were shown. The program adds

the new units given as data and no optimization of the plan is done. Several patterns are compared and analyzed and the least costly pattern is selected.

E. C. Henault, R. B. Eastvedt, J. Peschon and L. P. Hajdu (36) published a paper on power system long-range planning in the presence of uncertainty about future loads. They applied the stochastic dynamic programming to optimize a transmission system expansion in the presence of future loads variations.

In this thesis, a new approach for optimization of generation expansion patterns using the dynamic programming technique, is introduced. The effect of new unit size on both system reliability - spinning reserve, and cost of the expansion are studied.

## III. DATA REDUCTION PROGRAM

Since the six companies in the Pool have different sets of hourly load history data, it is necessary to organize these data to make them usable in the prediction of the future loads. The individual company data is recorded on different devices including magnetic tapes, cards, and disks. This makes it difficult to handle the data and necessitates the rerecording of all data on a common format.

In order to reduce the six sets of data to a common set, namely the "Pool Data Set", two methods were developed. Note that the amount of information contained in one data set exceeds 80,000 records; thus, approximately 500,000 records will be processed in order to obtain the Pool Data Set. This size of record cannot be processed efficiently by a simple search-and-store technique. A more efficient and speedy method should be used.

The first task is to store each individual set of data on a disk in such a way that, for each month of each year, the six company hourly loads are stored in a certain predetermined order. After storing the information, we can read these hourly loads from the disk and add six loads for the same hour of a given day, month and year. In the future, this could be done by transmitting the input data with a fixed format control and using the direct access input/output operations (37).

This problem was expedited by using a new efficient library routine called the IBM Sort/Merge Program (38) and the PL/I(F)

Compiler (39). This routine maintains the source program in storage throughout the compilation process, and successive phases of the compiler are passed against it. This means that the use of input/output data set is kept to a minimum, with a consequent improvement in performance. Since we are dealing with half a million records, any time saved, no matter how small, will result in a much larger saving when processing the entire data set.

Starting with the first company, a computer program was especially written for that company which reads the first data set, converts it to the common format for the Pool, and stores it on a scratch disk. This program is written in PL/I language. Next, the converted data set is sorted using the IBM Sort/Merge program after which it is stored on a permanent disk. The process is repeated for the remaining five companies. Then the six data sets on the permanent disk are merged using the same IBM program and the output is stored on the same disk. Finally, a small program is used to add the hourly loads for the six companies and store the so-called "Pool Data Set" either on tape or disk. A flow chart in Appendix A, Fig. A.1, illustrates the process in greater detail. A summary of C.P.U. times for the processing of the individual company data set is shown on Table A.1, while a computer listing of Program 1 named "Data Reduction Program" is shown at the end of the same appendix.

It should be also mentioned that before using any of the six sets of data, a duplicate set was prepared as insurance

against possible damage to the original data. This was done with the help of IBM Utilities program (40).

## IV. LOAD MODELING PROGRAMS

The load model is constructed by examining the statistical nature of the data at our disposal. To do this, the Pool Data Set is examined by a second computer program called "SUBROUTINE TREND" which picks the daily peaks for each day of each month. At the end of the month, the monthly peak load is determined and the mean of the daily peaks for that month is computed, considering the month to consist of 21 working days, i.e. neglecting the weekends and holidays. Next, the ratio of the mean to the peak for each month are computed. Finally, the ratio of the variance to the mean of daily peaks is calculated. Then the program picks the yearly peaks for every year and the process is repeated to the end of the available data set. The program also computes the total energy for each month and for each year.

This program is organized so as to analyze the Pool Data Set for pool as well as for the individual company data. This is done by setting special program "switches" to direct the program to provide the data needed. This gives more flexibility to the program and at the same time makes the program applicable for the Pool and any company of the Pool.

A flow chart for the computer program labeled as Program 2 is shown in Fig. B.1 in Appendix B. A summary of data for January is shown on Table B.1, while a summary of the monthly energy in MWHRS is shown on Table B.2 of that Appendix. The yearly energies in MWHRS versus the years, are shown in Fig. B.2.



A listing of Program 2 is shown at the end of the same Appendix.

After finding the arrays of the monthly peaks, the mean of daily peaks for the month and the yearly peaks, this data is fed to another part of the program which is designed to forecast the mean of daily peaks for each month using an exponential model (4). The program will project these given observations for many years to come.

So far, we have introduced two programs concerning load modeling, namely "Data Reduction" and "SUBROUTINE TREND". A third program was prepared to construct the load duration curves for each month of the year. These curves are required in evaluating the production costs of the existing system as well as the new generating capacity additions. This program is named "SUBROUTINE LOADUR". The programs add two consecutive hourly loads, compute the average value and divide this value by the monthly peak of that particular month. This means that we are dividing the day into 12 segments of 2 hours each. At the end of the month, we will have a series of these per unit loads (360 p.u. loads for a month of 30 days).

In order to construct the load duration curve, we have to find out the total time in p.u. within the month during which the load equaled or exceeded the load value under consideration (41). This is done by sorting all these per unit loads in decreasing order.

The mathematical analysis for load forecasting is given in Appendix C. A typical computer output for January is shown

in three tables in that Appendix. The mean of daily peaks in this month is forecast and the results are shown in Table C.1 with the 95% and 75% confidence belt limits. Table C.2 shows the result of forecasting the ratio of the mean and the monthly peak, while Table C.3 shows the results of forecasting the monthly peaks for the same month.

The flow chart for SUBROUTINE TREND is shown in Fig. C.1. The results for projecting the means of daily peaks in the month of January are plotted in Fig. C.2. The ratio of the means to the peak projections for the same month are plotted in Fig. C.3, while the monthly peak projections are shown in Fig. C.4 of Appendix C. The yearly peak forecasting is plotted in Fig. C.5. A listing of this subroutine is shown at the end of Appendix C where it is identified as Program 3.

A second array is computed to give the duration of all given per unit loads. This is done by considering the first per unit load in the load array of duration 2 hours. Then the duration of a load equal to that load or greater is still 2 hours and the per unit time of duration will be  $2/720$ . For the second per unit load, the duration of having a load equal to that load or greater will be 4 hours. In per unit, this will be  $4/720$  and so on for all loads. In plotting these per unit loads against their respective per unit time will have the so-called load duration curve. The area under that curve is the energy in per unit for that month. This curve will not be a smooth one, and it will be difficult to store all

these informations. Thus, an acceptable approximation will be preferable. This is done by fitting a straight line through these per unit loads using the least square method (42); in that case, we must store the slope of that line, the per unit load at  $t = 0$ , and the per unit load at  $t = 1$  per unit for each month. Now we store only three values compared to 720 values. This is repeated for every month of the year under study.

In order to obtain the megawatt load at any given duration of time, we multiply the per unit load at that given duration by the monthly forecasted peak for that month. So the load duration model in per unit can be used during the whole period of study.

It should be mentioned that the year 1967 was used for this computation because the Pool data for 1968 and 1969 are not available. However, since the model is in per unit, the deviations will be small. Also, the ratio of the monthly peaks to the yearly peak for 1967 was computed and stored. The method of construction of load duration curves is explained with a simplified example in Appendix D. The flow chart of the SUBROUTINE LOADURE is shown in Fig. D.3. The monthly load duration curves are shown in Fig. D.4 to Fig. D.15. The ratio of the monthly peaks to the yearly peak for 1967 for the Pool data is shown in Fig. D.16. A listing for that subroutine labeled as Program 4 is shown at the end of that Appendix.

## V. CAPACITY MODELING PROGRAM

In recent years, power system planning has become an increasing challenge as the electric systems have become progressively more complex. This makes it harder for the system planning engineer to design a system that is dependable and reliable. What do we mean by power system reliability? How can it be measured? We could say that power system reliability implies the uninterrupted supply of power in the right amount, in the right place, at the right time. This includes availability and system security. Availability is the probability that the power system will supply the load despite line or equipment outages and is measured by statistics which show what the response to such outages has been in the past. On the other hand, system security is a measure of the ability of the system to withstand stresses imposed by major accidents such as a loss-of-generation or transmission caused by malfunction or operator error (43). Our problem then is to organize the planning of a system such that it will be secure and able to supply power in the amount required. To do this, we need to find some mathematical laws which enable us to test our system reliability.

Assume that we have a single 20MW generator, for example. Suppose that this generator has a 2-day forced outage every 100 days. We could say that the probability of having the 20MW available is  $q = \frac{100-2}{100} = 0.98$ , and the probability of having zero MW (i.e. the generator is out of service), will be

$$p = \frac{2}{100} = 0.02 \quad (5.1)$$

The duration of that outage is 2 days and the interval of occurrence is 100 days, so we compute the probability of outage as

$$p = \frac{\text{duration}}{\text{interval}} \quad (5.2)$$

The frequency of occurrence  $F$  is computed as

$$F = \frac{1}{\text{interval}} = \frac{\text{probability}}{\text{duration}} \quad (5.3)$$

Now we could say the availability of the 20MW generator is 98% but the security is zero, because the generator will be out of service and the load will be completely interrupted for 2 days in every 100 days as mentioned before. Thus, the supply of power is not continuous and the system is "insecure".

Actually, a power system is much more complicated than our simple example. Even small systems have several generators and larger systems such as the Iowa Pool will have a large number (there are over 70 generators in the Iowa Pool). In such systems, it is much more difficult to measure the availability and assure the security.

The application of probability theory in electric utility industry was begun in the 1920's. Much interest was aroused by T. C. Fry (44) in 1928. In 1933, Lyman (9) of Dusquesne, and Smith (10) of Public Service Company of New Jersey, suggested using probability theories to evaluate the spinning reserve requirements. Forbes and Bellows, of Consolidated Edison, presented a paper in Toronto in 1941 on this subject.

H. P. Seelye (13) presented another paper in 1947, in which the interval in years between forced outages of various magnitudes was computed. Calabrese (45), Loane (46) and Watchorn (47) developed methods which combine the probability of outage with peak load duration to evaluate the probability that the load might exceed the available capacity. Calabrese (48) continued his computation and determined the Kwhr of load that might be interrupted. Then he computed an index of reliability which related the probable loss of energy to the system size. Adler and Miller (49) published a paper in 1946 similar to Seelye's approach.

In general, there are three different methods that measure the system reliability:

1. Loss of Load Probability (LOLP).
2. Loss of Energy Probability (LOEP).
3. Loss of Capacity Probability (LOCP).

Appendix E gives detailed discussion of all three methods including a mathematical derivation. These three methods can provide a measure of system reliability. In order to evaluate the probability of the system's failure to carry the load during any period (such as a day, a month or a year) it is necessary to determine the probability of failure at each instant during that period and then integrate these probabilities to find the cumulative or total probability of failure for the particular period under study.

To find the probability that the load will exceed the installed capacity less maintenance outages and forced outages,

we must first decide what value of static reserve or margin we would allow such that the index of reliability is below a predetermined value. In Appendix E, several mathematical models to compute the index of reliability or the risk index are explained.

A computer program was written to compute the LOLP for the Pool and at the same time to compute the LOCP and compute the spinning reserve for every forecasted load. It will also compute the production costs for the expansion patterns which will be explained in Appendix G later. The program is labeled Program 5, which is very flexible and fast. It can handle up to 200 units and it takes care of forecasting errors, if necessary, in computing the index of reliability. A complete flow chart for the computer program is shown in Fig. E.11. Also, the megawatt outage, the probability of having such an outage, the duration of this outage, and the interval in days between two successive outages are given in Table E.15. The available capacity and the cumulative probability of the Pool as of January 1970 is shown in Table E.16 of the same appendix. The probability and the intervals in days versus the megawatt outage for the Pool is drawn on a semilogarithmic paper as shown in Fig. E.9. The cumulative probability versus the megawatt outage is shown in Fig. E.10.

A computer program listing for Program 5 is shown in Appendix E.

## VI. INFLUENCE OF GENERATOR SIZE ON THE RELIABILITY AND THE COST OF POWER SYSTEM

In this chapter, we will discuss the effect of the size of the new added units on the reliability of the system, the spinning reserve and the investment cost. It is sometimes claimed that the larger the unit size, the more economical it will be. If one examines only the efficiency or the cost per kilowatt of installed capacity, this may be true. Our problem, however, is to minimize the total cost of owning and operating the system while maintaining a given reliability index. Thus, there are factors other than unit efficiency and cost per kilowatt that are important.

One such factor is reserve. Any system, if it is to be operated at a reasonably high reliability index, must have reserve capacity to act as back-up in the event of forced (unplanned) unit outages. This implies that the reserve capacity must either be installed or must be purchased, probably at a rather high rate. Reserve capacity is affected by three factors, all of which require investment. These are unit size, forced outage rate and maintenance time. But all three are a function of unit size.

To serve a given level of system load, the larger the size of generating units added, the larger the reserve requirement needed to maintain a constant index of reliability. This is because enough reserve must be available to back up loss of the largest unit. Similarly, adding units with high forced



outage rate increases the spinning reserve requirement. Since larger units generally have higher forced outage rates, this factor is size dependent in favor of smaller units. Finally, reserve is needed to cover planned maintenance time and since these outages are greater for large units, this factor also favors smaller units. These increases in reserve for any of the three factors mentioned above - size, higher forced outage rate or longer maintenance period - all require additional investment. However, the length of time required for scheduled maintenance, lies entirely within the control of electric utility management and it can be reduced by providing enough manpower and equipment to decrease that time. Also, the forced outage rate could be controlled to some extent by better design, materials, workmanship, erection and the use of skilled operating practices.

The problem of higher reserves required by the remaining factor, unit size, can also be controlled by selecting the unit size on an economical basis such that the cost of the reserve penalty required by the unit size is kept to a minimum. For example, in a given power system, we might install a 1,000-MW unit or, alternatively, ten 100-MW units to carry a 1,000-MW load. Emergency outage of the single 1,000-MW unit (with a probability of occurrence equal to 6.5%) would be far more damaging to the system than loss of one or two of the 100-MW units. In the first case, the reserve required will be 1,000-MW while in the second it is about 200-MW. Also, the

index of reliability for the second system will be much higher than that of the first system. The investment cost of the 1,000-MW unit will be less than that for ten 100-MW units, but this advantage is cancelled by the cost of the spinning reserve which is 5 to 1. In this example, then, we can say without hesitation that the ten 100-MW units are more economical than the one 1,000-MW unit. This would still be true even if we minimize the forced outage rate and the maintenance periods. The reason is that the spinning reserve necessary to give the system any degree of reliability for the 1,000-MW unit should be 1,000-MW which is not practical and is clearly uneconomical.

For future planning of generating additions, we have to assign a certain degree of reliability for the system under study. This choice is very important and it affects the expansion costs and the size of the new units drastically.

A one day of insufficient capacity for every 5 years<sup>2</sup> (risk index =  $1/1260 = 0.0007936$  and an index of reliability =  $0.9992063$ ) might be accepted as the threshold risk level. The selection of the reliability index depends on the importance of the loads supplied, the type of loads, and the locations of these loads. For most systems in the U.S.A., a one-day outage for every 10 years (risk index =  $1/2520 = 0.0003968$  and an index of reliability =  $0.9996032$ ) is considered adequate for planning purposes.

---

<sup>2</sup>Here we consider a "year" as 12 months of 21 "working" days, or 252 days.

Now, having selected the reliability index level, the planner must measure the existing system reliability using any method convenient to him. Should the computed risk index be higher than the preselected value, a new generator addition (or purchase contract) should take place. The next question is what size unit should be added. This is not an easy question to answer and further investigations will be required before a decision can be made. Usually, several expansion plans will be studied and the most economical one chosen. As an example for this procedure, for the Iowa Pool System, the index of reliability was measured by two methods. Both methods indicated the need to add new generating capacity. For comparison, two plans were then studied. One unit of 400-MW was added to the system versus 2 units of 200-MW each and the costs, the spinning reserve, and the risk index were calculated and compared. This is shown in Appendix F. The spinning reserve for the original system & the system with 400 MW are shown in Fig. F.4. In this appendix, effect of the new unit size on the reliability and the spinning reserve of the Pool system is discussed and a mathematical method is introduced to select the size of the new unit added to the system in such a way as to decrease the spinning reserve and increase the capacity factor of that unit. Three expansion patterns are shown and the effect of the size of each unit in any pattern on both the reliability and the spinning reserve is shown in Fig. F.5 to Fig. F.10.

## VII. COST OF EXPANSION PATTERNS

With the aid of the previous programs, including the LOLP or LOCP routines, units are added to the system subject to the constraint that the reliability index must not be exceeded. Then another program will compute the present worth of all the costs over the period under study.

The cost of expansion during any year consists of five components (21); investment for generation, investment for interconnection capacity, investment for the transmission network, the operation and maintenance costs, and the cost of fuel. The sum of these five components will give the amount of money spent during that year to carry out the expansion in the year under consideration. For our study, we do not consider the transmission cost and our total cost will exclude that part in computing the cost of expansion.

After computing the cost of all alternatives during that period of years, the program moves to the next period and computes the total costs for all the alternative patterns during that period and so on till the end of the planning period under study. Then, with the help of dynamic programming, which will be explained later, the choice of the optimum expansion is completed. The data required for the cost program consists mainly of generation data as follows:

1. A plot (or table) of annual investment cost for generation in dollars per KW capacity (see Fig. G.1 and G.2, Appendix G). This is done by approximating

the investment cost for generation as a function of unit size by an exponential approximation (21). Let this relation be in the form

$$D_G = K_G \cdot e^{-L_G \cdot \text{CAP}} \quad (7.1)$$

where  $D_G$  is the annual investment cost for generation in dollars per kW and  $K_G$  and  $L_G$  are two constants computed by the least square method in fitting the exponential model. CAP = unit capacity in kW.

2. A plot (or table) of annual operation and maintenance costs in dollars per kW capacity (see Fig. G.3 and Fig. G.4). This is also done by approximating the cost as a function of unit size by an exponential model as follows:

$$D_{OM} = K_{OM} \cdot e^{-L_{OM} \cdot \text{CAP}} \quad (7.2)$$

where  $D_{OM}$  = annual operation and maintenance costs in dollars per kW.

$K_{OM}$  and  $L_{OM}$  are two constants which define the exponential model.

3. The annual fixed charge rate ( $x$ ) computed as an annual cost, is expressed as a percentage of the total plant investment (10).
4. Availability factor ( $F_C$ ) of all units, new or old.
5. Fuel costs  $D_F$  for each unit, new or old, in cent/ $10^6$  Btu's.

6. The heat rate (H) of each unit, new or old, in Btu/kW-hr.
7. The maximum capacity (CAP) of all units, new or old, in Mw.
8. The interest rate (i) on money.
9. The inflation constant (f), that is the annual rate at which money devalues in an inflationary period.
10. The cost of transmission lines ( $D_T$ ) if any, in k dollars. The computation of the different components will be performed in the following steps:

Assume  $Y_0$  to be our starting year for computation.

1. For all generators in the existing system, calculate the monthly fuel cost ( $M_F$ ) in k\$ if all units are operated continuously at full load.

For unit j, we will have

$$M_{F_j} = (H_j) (D_{F_j}) (CAP)_j (7.3 \times 10^{-6}) \text{ k } \$ \quad (7.3)$$

where

$M_{F_j}$  = monthly fuel cost in thousands of dollars for the j th unit of the original system.

$H_j$  = heat rate of the j th unit in Btu per Kw-hr

$D_{F_j}$  = fuel cost for the j th unit in year  $Y_0$ , in cents per million Btu

$CAP_j$  = full load net capability of the  
j th unit in Mw

730 = total hours in the month (= 8760/12)

The  $M_{Fj}$  will be computed for every unit in the original system in the study.

2. Compute the annual operation and maintenance cost for each unit for the year  $Y_0$ . This is done easily using equation 6.2, and we have

$$D_{OM_j} = (K_{OM} e^{-L_{OM} \cdot CAP_j}) \cdot CAP_j \quad \text{k\$} \quad (7.4)$$

where  $D_{OM_j}$  = annual operation and maintenance cost for the j th unit in thousands of dollars

$K_{OM}$  and  $L_{OM}$  as defined before

$CAP_j$  = full load net capability of the  
j th unit in Mw

The total annual operation and maintenance cost ( $A_{OM}$ ) in thousands of dollars will be given as

$$A_{OM} = \sum_{j=1}^N K_{OM} (e^{-L_{OM} \cdot CAP_j}) \cdot CAP_j \quad \text{k\$} \quad (7.5)$$

where  $N$  = the total number of units in our system.

3. All costs are based on the initial year  $Y_0$ . In order to consider the effect of inflation, we have to assume that the cost of any item

in year Y is  $(1+f)$  times the cost of the same item in year Y-1. For simplicity, we will consider  $f$ , the inflation factor is the same for generation, fuel, and operation and maintenance costs, i.e.

$$\begin{aligned} (\text{costs at year } Y_m) &= (1+f)^m (\text{costs} \\ &\text{at year } Y_0) \end{aligned} \quad (7.6)$$

where  $m$  = number of years beyond  $Y_0$

If  $Y_0 = 1970$ , then we should know the costs at  $Y_{S-1} = 1969$ , the year before the beginning of the expansion program and define

$$F_L = (1+f)^{(Y_0 - Y_{S-1})} \quad (7.7)$$

4. Compute the total annual cost of operation and maintenance in the year  $Y_{S-1}$ , i.e.

$$(A_{OM})_{Y_{S-1}} = (A_{OM})_Y (F_L) \quad (7.8)$$

5. For any year Y the inflation factor will be

$$F_Y = (1+f)^{Y - Y_0} \quad (7.9)$$

6. Once we select the size of the new added capacity using a constant reliability index, we have to compute the investment cost for this new added capacity. Using equation 6.1, we can write the investment cost of a new unit to be added as

$$D_G = K_G (e^{-L_G \cdot C_Y}) (C_Y) \quad \text{k\$} \quad (7.10)$$



where

$D_G$  is the investment cost in k\$

$K_G$  and  $L_G$  are two constants as before.

$C_Y$  is the new unit capacity in MW.

Now if the year in which the present dollar value is to be calculated is  $Y$ , the new cost will be

$$(D_G)_Y = D_G \cdot F_Y \quad \text{k\$} \quad (7.11)$$

For any year with no generation addition, this term is set to zero.

7. The annual fixed charge cost  $(A_x)$  will be computed as

$$(A_x)_Y = (D_G)_Y \cdot \frac{x}{100} \quad \text{k\$} \quad (7.12)$$

where

$(A_x)_Y$  = annual fixed charge cost at year  $Y$ , in  
k \$

$D_{G_Y}$  = the investment cost in k\$

$x$  = annual fixed charge rate in percent.

8. Compute the cost of purchase energy ( $D_p$  in k\$) if necessary.
9. Compute the annual operation and maintenance for the new unit added to the system. This is done by using equation 7.2 with the result

$$(D_{OMN})_Y = (K_{OM} e^{-L_{OM} \cdot C_Y}) \cdot C_Y \cdot F_Y \quad \text{k\$} \quad (7.13)$$

where  $D_{OMN}$  = the cost of operation and maintenance for the new unit in k\$.

10. Compute the monthly fuel cost for new addition

$$M_{FN} = (H) (D_F) (C_Y) (7.3) \times 10^{-6} \quad \text{k\$} \quad (7.14)$$

where  $M_{FN}$  = monthly fuel cost for new unit.

11. Compute the total operating and maintenance costs ( $A_{TOM}$ ) which will be given as:

$$(A_{TOM})_Y = (A_{OM})_Y (F_Y) + (D_{OMN})_Y \quad \text{k\$} \quad (7.15)$$

In the previous calculations, we have assumed all units to be in service at full load and computed the fuel cost for this condition. This is usually not the case, so we must develop a means of correcting for this unrealistic assumption.

Two methods could be used:

- (a) Economically dispatch all units of the system every hour and compute the hourly fuel costs for each unit on service. This method is quite accurate but requires a great deal of computation since each year has 8,760 hours.
- (b) Using the monthly load duration curves computed before. Using a straight line approximation for load duration, we may calculate the capacity factor for all units that are required to supply the peak load. For

example, a typical load duration curve is shown in Fig. 7.1.

The straight line approximation is characterized by two parameters:

- (1) Load =  $y_1$  in per unit based on the monthly peak (the forecasted monthly peak,  $P_m$ )  
 $= y_1 P_m$  Mw at  $t=0$
- (2) Load =  $y_2$  in per unit  
 $= y_2 P_m$  Mw at  $t=1.0$

In any month, we consider  $N$  units including the new unit to be added. Assume an availability factor of about 0.9 to allow for scheduled and forced outages.

Let the largest unit (or the most efficient unit) be placed at the bottom of the load duration curve. Let this unit have a capacity  $C_1$  Mw. If this unit is a mature unit, that is, it has completed its initial break-in period, then use 0.9 as the availability factor. If this unit is a new one, the availability factor should be computed using the risk method, i.e.

$$F_{C_n} = \frac{\text{effective capability of that unit}}{\text{the size of the unit}} \quad (7.16)$$

where  $F_{C_n}$  = the availability factor for the new unit. This will be discussed in Appendix F.

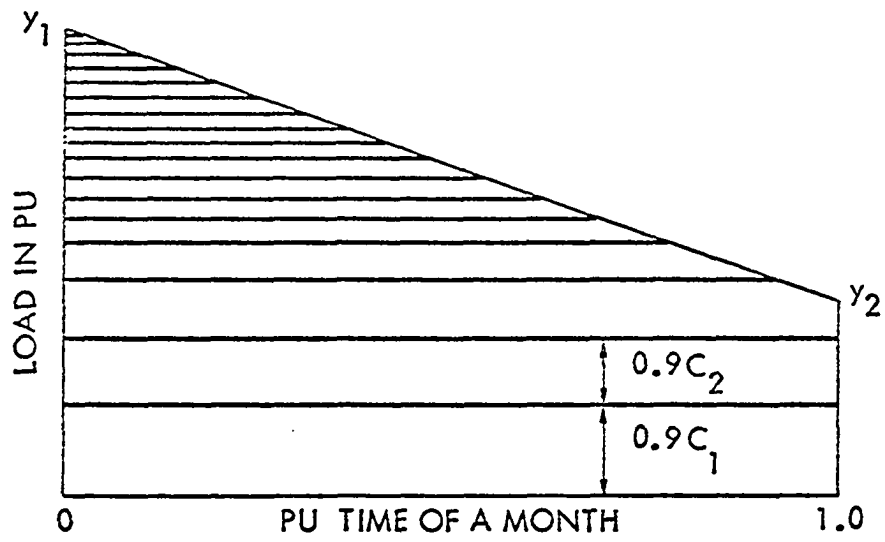


Fig. 7.1. Monthly load duration curve

Suppose that the first unit with capacity  $C_1$  Mw is mature. Then the energy contribution for that unit will be as shown in Fig. 7.1. This energy is the area  $(0.9C_1 \times 1.0)$  Mw. The second unit in size will be added "on top" of the first one and the computation continues. It is quite possible that some of the small units will have zero capacity factor. This process will provide capacity factors for all units necessary to supply the peak load for the period represented by the load duration curve. Let  $F_{C_j}$  be the capacity factor of unit  $j$ . The total annual fuel cost  $(A_{TF})$  will be given as the sum of 12 months, as:

$$(A_{TF})_Y = \sum_1^{12} \sum_{j=1}^{R_j} (F_{C_j}) (M_{F_j}) (F_Y) \quad (7.17)$$

where  $R_j$  is the number of units in service in a given month to supply the peak load.

12. Now to compute the total annual cost for year  $Y$   $(A_T)$ . This is the sum of all the previously computed costs, i.e.

$$(A_T)_Y = (D_G)_Y + (A_{TOM})_Y + (A_{TF})_Y + (A_X)_Y \quad (7.18)$$

13. The total annual energy for year  $Y$  in mills per kilowatt-hour  $(A_E)$  will be given as:

$$(A_E)_Y = (A_T)_Y / (L_Y) (F_{LD}) \quad (8.76)$$

$$\text{mills/kw-hr} \quad (7.19)$$

where  $L_Y$  = the yearly peak in kilowatt and

$F_{LD}$  = the annual load factor for year Y

13. Now calculate the  $(PW)_Y$ , the present worth of the expansion program that is carried out during Y. Let  $i$  be the interest rate and  $Y_{PW}$  be the year for which the present worth is required.

Then

$$(F_{PW})_Y = \frac{1}{(1+i)^{Y-Y_{PW}}} \quad (7.20)$$

and the  $(PW)_Y$  will be given as

$$(PW)_Y = (A_T)_Y (F_{PW})_Y \quad (7.21)$$

Note that for every interval of study, we will have more than one alternative which will satisfy the reliability index. We must store all these  $(PW)_Y$  factors that will result. The choice of the optimum expansion pattern will be discussed later.

The computer program listing is shown on Appendix E as mentioned before. The fuel cost for each unit of Iowa Pool system at full load in k\$ is shown in Table G.1, while the operation and maintenance costs for the same units are shown in Table G.2. The capacity factors for those units are shown in Table G.3.

In Table G.4, the production costs as well as the new addition costs for year 1971 are shown.

The same costs for year 1972 are shown in Table G.5 and Table G.6.

VIII. OPTIMIZATION OF EXPANSION PATTERNS'  
COSTS USING THE DYNAMIC PROGRAMMING TECHNIQUE

Bellman's so-called "dynamic programming" (2), (50) has gained considerable popularity in recent years as a complement to the other classical methods of optimization. Dynamic programming attains its greatest practical significance in conjunction with the modern digital computer. The essential feature of dynamic programming is that it reduces the N-stage decision process to a sequence of N single-stage decision processes (51). This enables us to solve our problem in a simple iterative manner using a computer of moderate size. This is done by the use of the fundamental principle of optimality:

A policy which is optimal over the interval  $o \rightarrow N-1$  is necessarily optimal over any subinterval  $u \rightarrow N-1$ , where  $o < u < N-1$ .

A simple example will demonstrate the basic idea of dynamic programming and shows its applicability in optimizing the expansion patterns costs.

A truck driver is going from city I to city II. The number of stages he has to travel are known to be 7. The increments in criterion functions between different stages are shown in Fig. 8.1.

These increments are cost functions for the highways, including gas, lodging, allowance, repair costs (if any), and food. The problem now is to find the path that will optimize the total criterion function between I and II. The number of



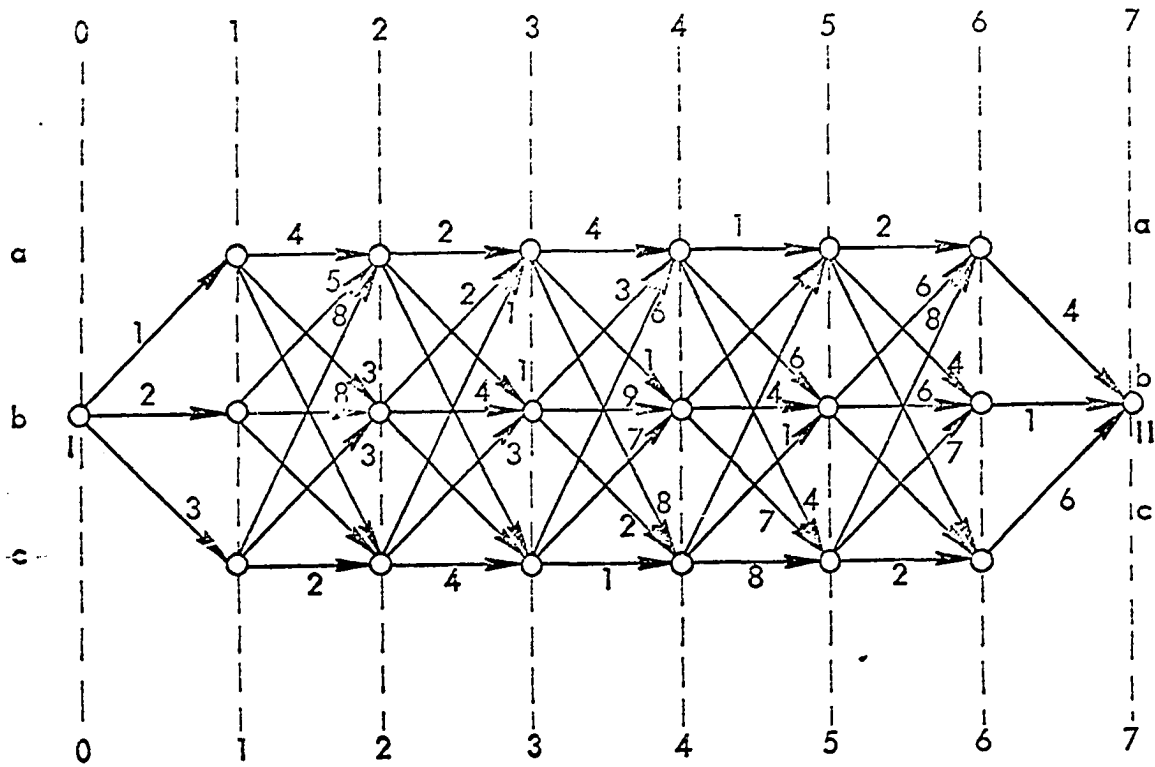


Fig. 8.1. Dynamic programming example

stages  $N$  is equal to 7. Let the number of possibilities at any stage  $r=3$ , and the dimensionality of the problem  $n=1$  since we have only one source node (I) and one sink node (II).

The total number of combinations  $T$  to choose from is given by the following equation:

$$\begin{aligned} T &= r^{n(N-1)} \\ T &= 3^6 = 729 \end{aligned} \tag{8.1}$$

Instead of trying all those possible combinations, we proceed as follows:

Step 1: Going from stage 0 to stage 1, store all the increments as shown by the numbers 1, 2 and 3 with asterisks in Fig. 8.2a. Note that we do not know at this time if the optimal policy will take us through the states  $a_1$ ,  $b_1$ , or  $c_1$  so we store all of them.

Step 2: Going from stage 1 to stage 2. Here we compute all combinations from  $a_1$  to  $a_2$ ,  $b_2$  and  $c_2$  respectively. The same will be done w.r.t.  $b_1$  and  $c_1$ . Then retain the minimum cost of the (3 x 1) array at  $a_2$ ,  $b_2$  and  $c_2$  and neglect the other values. Suppose that these minima are 5, 4 and 5 respectively, as shown in Fig. 8.2b. Now we can say that the optimum path will surely pass through state  $b_2$ . Thus, we also know that we arrive there from state  $a_1$ .

Step 3: In this step, we move from  $a_2$  to  $a_3$ ,  $b_3$  and  $c_3$

and store the minimum value as shown in Fig. 8.2c. Similarly, from  $b_2$  to the same three states and for  $c_2$  also. In state  $a_3$ , we have two values which are equal, so we must store both of them.

Step 4: Going from stage 3 to stage 4, we repeat the same process with the results shown in Fig. 8.2d.

Step 5: Again, from stage 4 to stage 5, we find three minima as shown in Fig. 8.2e.

Step 6: Going from stage 5 to stage 6, we compute the minima of Fig. 8.2f.

Step 7: Finally, going from step 6 to step 7, we compute the minimum cost shown in Fig. 8.2g.

The optimum path will be shown on Fig. 8.3.

The total cost is computed to be 14. A summary of calculations in the previous steps are shown in Table 8.1. The number of combinations (calculations) will be

$$\begin{aligned} &= 2r + (N-2)r^2 \\ &= 6 + 5(9) = 51 \end{aligned} \tag{8.2}$$

compared to 729 combinations given by equation 8.1.

This example shows the dynamic programming technique and shows clearly the advantage of reducing the number of combinations to choose the optimum path. The technique explained before is called "Forward Dynamic Programming".

Another way to solve the previous problem is to start from the sink node (II) and go backward to the source node (I).

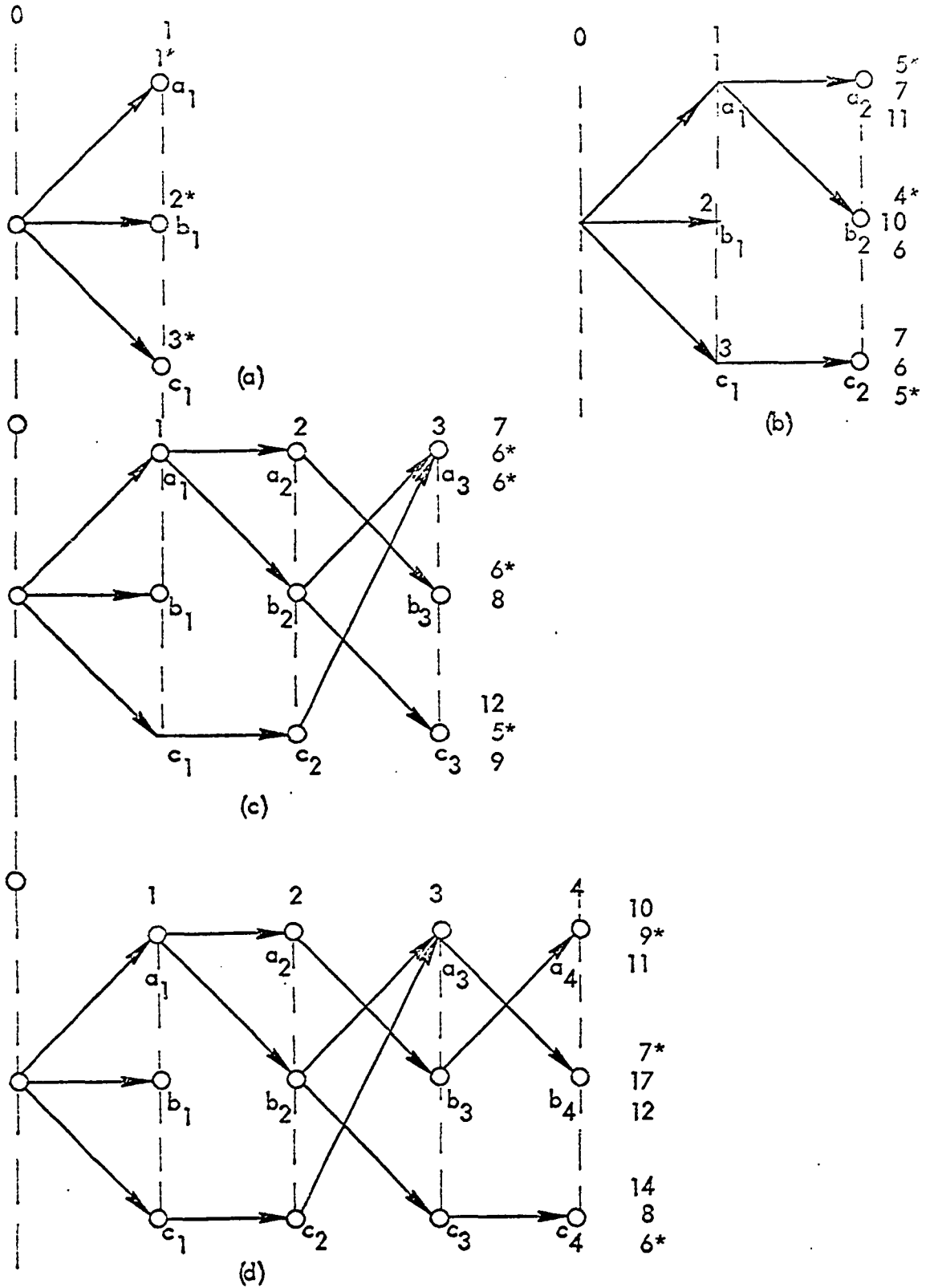


Fig. 8.2. Steps of dynamic programming example

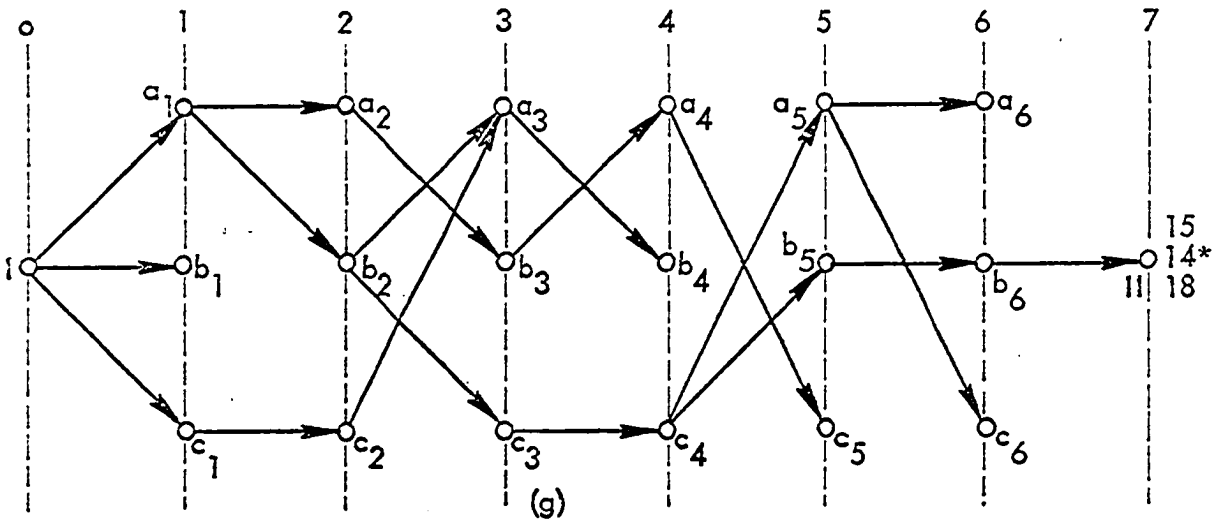
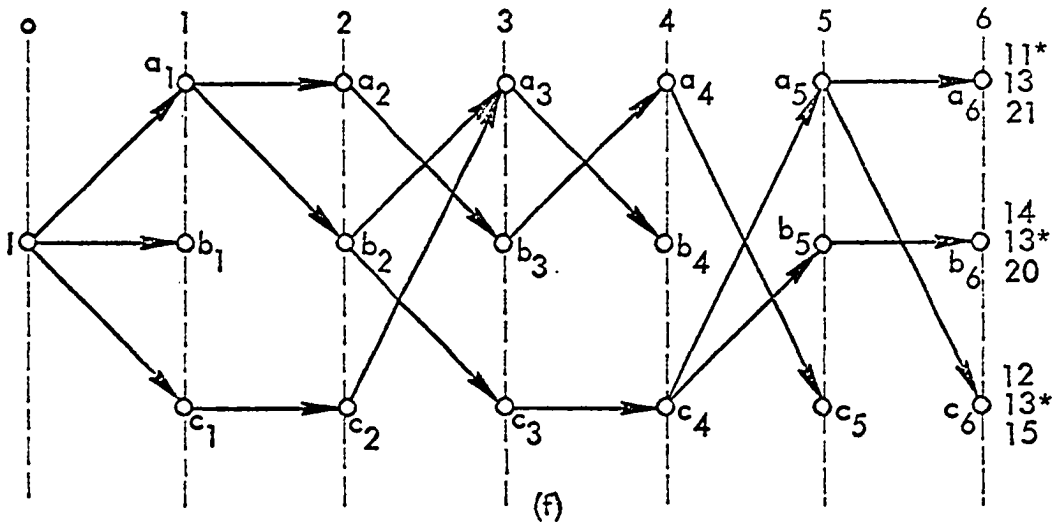
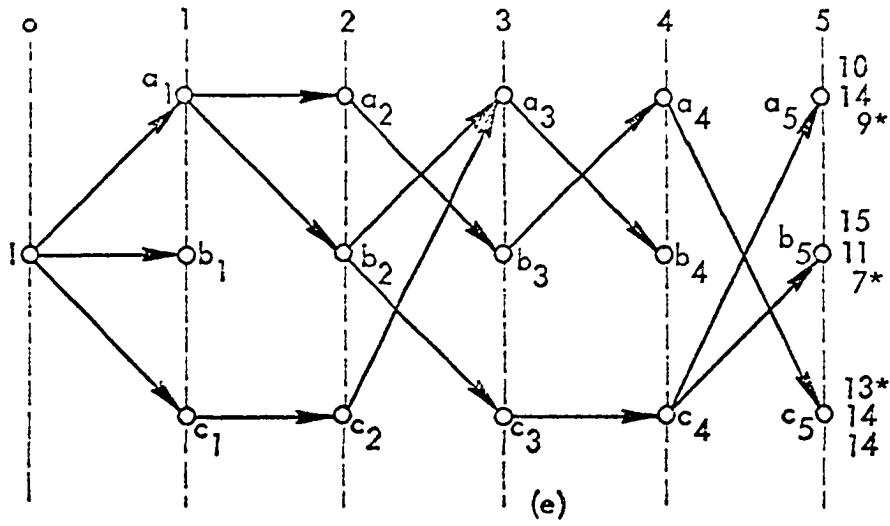


Fig. 8.2 (Cont.)

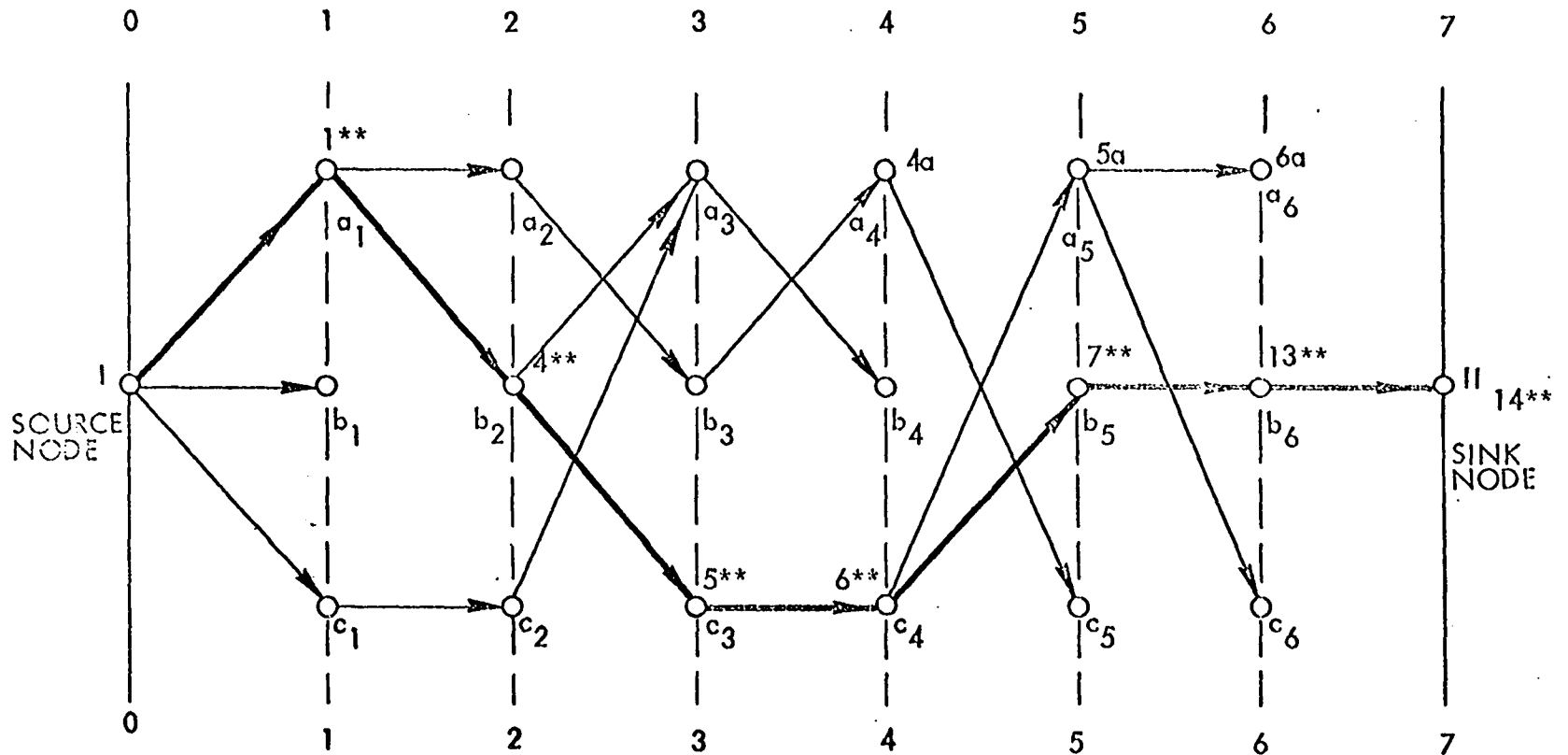


Fig. 8.3. The optimum path between source and sink nodes

Table 8.1. Summary of calculations done in various steps

Stages	1			2			3			4		
States	<u>a</u>	<u>b</u>	<u>c</u>	<u>a</u>	<u>b</u>	<u>c</u>	<u>a</u>	<u>b</u>	<u>c</u>	<u>a</u>	<u>b</u>	<u>c</u>
a	1**			5*	4**	7	7	6*	12	10	7*	14
b		2*		7	10	6	6*	8	5**	9*	17	8
c			3*	11	6	5*	6*	8	9	11	12	6**

Table 8.1 (Cont.)

Stages	5			6			II		
States	<u>a</u>	<u>b</u>	<u>c</u>	<u>a</u>	<u>b</u>	<u>c</u>	<u>a</u>	<u>b</u>	<u>c</u>
a	10	15	13*	11*	13**	12*	15		
b	14	11	14	13	14	13		14**	
c	9*	7**	14	20	20	15			18

\* or \*\* states to be retained through which possible optimum path candidates may pass

\*\* states in the optimum path

This could be done in our example since the cost functions for the highways are reciprocal, i.e. the cost is the same for both directions since it depends on the mileage and the condition of the highway. This technique is called "Backward Dynamic Programming". In power system planning, we can not apply the backward programming because every stage will depend on the preceding one.

Assume that the source node (I) is the initial state of the power system. Let the informations of the initial state such as the installed capacity, the load demand, the reliability index, and the initial year of planning be known.

At that state, we assume that the system reliability index is less than the predetermined value and a new addition should be made. Assume that during the initial stage, three different patterns are added to the system which means that three states are created at the first stage. The branches between the initial state and the newly-formed three states represent the overall costs of the system including the production cost during that stage, the operation and maintenance cost, and the investment cost of the new additions with the fixed charges all based on 1970 prices.

Assume that another three patterns are added to each state of the first stage, we will create another three states at the second stage, and the number of branches between the two stages will be equal to nine. These branches will represent costs based on 1970 prices as before. The process will continue



till the end of the period of planning. We will then have a flow diagram similar to Fig. 8.1 which will represent all the stages.

Using the "Forward Dynamic Programming" as before, we could select the optimum path that takes us from the initial state to the final state and at the same time will optimize the cost of the expansion pattern. This path will be indicated by the states through which it passes together with the branches which constitute segments of this path. The states will indicate the time of the additions, their size, and the type of the new additions while the branches will indicate the costs of those additions. The only difference between this configuration and that given in the example shown before, is that at any state of any stage, the reliability index, as well as the overall cost, will depend on the states at the previous stages and this should be checked before moving to the next stage. The application of this technique is shown in Appendix G.

So far, we have not considered the selection of the locations of the new additions and the transmission addition costs associated with that choice. This could be done by considering all possible available locations and compute the new transmission addition cost and other costs which depend on those locations. These costs are added to each branch costs.

For example, if we assume that four different locations are available at the first stage of the previous example, the

additional costs associated with those locations will be computed.

Adding these costs to the cost of the first branch, we will form four first states instead of one, each representing a different location. Also, another four states will be formed instead of the second state, and the same will occur for the third state. Thus, twelve states will be generated at the first stage, compared to just three states as before.

This will continue for the second stage and the number of states will be equal to 144 states. Equation 8.1 can now be rewritten as

$$T = (r.w)^{n(N-1)} \quad (8.3)$$

where

$r$  = the number of patterns possibilities at any stage,

$w$  = the number of locations available at any stage,

$n$  = dimensionality of the problem (equal to the number of source nodes and in our case is equal to 1),

$N$  = number of stages, and

$T$  = total number of combinations

In the previous example, if we let  $w$  equal 4, then  $T$  will be equal to  $12^6$  which is a very large number and the problem should not be attempted, even by digital computer, due to storage size and speed limitations. However, using the forward dynamic programming, the number of combinations will reduce to

$$T = 2rw + (N-2)r^2w \quad (8.4)$$

which in our example with  $w$  equal 4 results in 204 combinations compared to  $12^6$ . This problem could be solved by a small or medium-size computer.

## IX. CONCLUSION

The dynamic programming technique is a powerful method that may be used in the optimization of power system long-range planning. Its great practical significance with modern digital computers which accept only discrete data or "data sequences", makes it applicable to solve problems that cannot be optimized by classical methods. Moreover, it reduces the multi-stage decision process to a sequence of single-stage decision processes enabling us to solve the problem in a simple iterative manner. In this investigation, dynamic programming was used to select the generation expansion pattern that will optimize the overall costs and at the same time satisfies the reliability constraint imposed on the system to ensure both availability of power and system security. The size of the new unit added to the system has great impact on the reserve capacity to act as a backup in the event of forced outage and on the economy of the system. Considerable care should be given in selecting that size. The choice of the unit size is measured by how much reserve it requires to maintain the same level of system reliability in addition to its economical features. This choice depends on many factors, such as the system size, the designed reliability index and the system interconnection with other systems. A unit which appears to be an uneconomical choice in a certain stage could be a part of the optimum expansion pattern obtained by dynamic programming. Also, we could easily locate the new

units added in such a way so as to optimize the overall costs of the system including the transmission needs using the same technique.

## X. LITERATURE CITED

1. Elgerd, O. I. Control systems theory. New York, N.Y., McGraw-Hill Book Co., Inc. 1967.
2. Bellman, R. E. and Dryfus, S. E. Applied dynamic programming. Princeton, New Jersey, Princeton University Press. 1962.
3. Denn, M. M. Optimization by variational methods. New York, N.Y., McGraw-Hill Book Co., Inc. 1959.
4. Hore, R. A. Advanced studies in electrical power system design. London, Chapman and Hall Ltd. 1966.
5. Atkinson, H. M. Alternating current generation and distribution - changes contemplated in Atlanta. American Electrician 11, No. 7: 333-334. 1899.
6. Bailey, R. Fundamental plan of power supply in the Philadelphia area. AIEE Transactions 49: 605-620. 1930.
7. Powell, R. C. Steam power development of the Pacific Gas and Electric Company. AIEE Transactions 50: 55-60. 1931.
8. Fugill, A. P. I-Combined reliability and economy in operation of large electric systems - The Detroit Edison Company. AIEE Transactions 51: 859-865. 1932.
9. Lyman, W. J. Fundamental consideration in preparing a master system plan. Electrical World 101: 788-792. 1933.
10. Smith, S. A., Jr. Spare capacity fixed by probabilities of outages. Electrical World 103: 222-225. 1934.
11. Smith, S. A., Jr. Service reliability measured by probabilities of outage. Electrical World 103: 371-374. 1934.
12. Lyman, W. J. Calculating probability of generating capacity outages. AIEE Transactions 66: 1471-1477. 1947.
13. Seelye, H. P. Outage expectancy as a basis for generator reserve. AIEE Transactions 66: 1483-1488. 1947.
14. Waring, M. L. Planning the development of a metropolitan electric system. AIEE Transactions 67, Part 2: 1467-1473. 1948.

15. Williams, A. L. and Kanouse, E. L. Power system planning in the city of Los Angeles. AIEE Transactions 69, Part 2: 400-408. 1950.
16. Kirchmayer, L. K., Mellor, A. G., O'Mara, J. F. and Stevenson, J. R. An investigation of the economic size of steam-electric generating units. AIEE Transactions on Power Apparatus and Systems 74, Part 3: 600-614. 1955.
17. Steinberg, M. J. and Cook, V. M. Evaluation of unit capacity additions. AIEE Transactions on Power Apparatus and Systems 75, Part 3: 169-179. 1956.
18. Kirchmayer, L. K., Mellor, A. G. and Simmons, H. O., Jr. The effect of interconnections on economic generation expansion patterns. AIEE Transactions on Power Apparatus and Systems 76, Part 3: 203-214. 1957.
19. Pitcher, W. J., Kirchmayer, L. K., Mellor, A. G. and Simmons, H. O., Jr. Generator unit size study for the Dayton Power and Light Company. AIEE Transactions on Power Apparatus and Systems 77, Part 3: 558-563. 1958.
20. Brennen, M. K., Galloway, C. D. and Kirchmayer, L. K. Digital computer aids economic - Probabilistic study of generator systems - I. AIEE Transactions on Power Apparatus and Systems 77, Part 3: 564-571. 1958.
21. Galloway, C. D. and Kirchmayer, L. K. Digital computer aids economic - Probabilistic study of generator systems - II. AIEE Transactions on Power Apparatus and Systems 77, Part 3: 571-577. 1958.
22. Baldwin, C. J., Gaver, D. P. and Hoffman, C. H. Mathematical models for use in the simulation of power generation outages: I - Fundamental considerations. AIEE Transactions on Power Apparatus and Systems 78, Part 3B: 1251-1258. 1959.
23. Baldwin, C. J., Billings, J. E. and Gaver, D. P. Mathematical models for use in the simulation of power generation outages: II - Power system forced outage distributions. AIEE Transactions on Power Apparatus and Systems 78, Part 3B: 1258-1267. 1959.
24. Dillard, J. K. and Sels, H. K. Program for planning. Electric Light and Power 37: 50-52. February 15, 1959.

25. Reps, D. N. and Rose, J. A. Program for planning, Part II - Game theory techniques - A new economic tool. *Electric Light and Power* 37: 56-62. May 15, 1959.
26. Williams, J. D. *The compleat strategist*. New York, N.Y., McGraw-Hill Book Co., Inc. 1954.
27. Baldwin, D. J. and Hoffman, C. H. Program for planning, Part III - Evaluating chance in planning. *Electric Light and Power* 37: 55-57. August 15, 1959.
28. Dale, K. M., Ferguson, Hoffman, C. H. and Rose. Production cost calculations for system planning by operational gaming models. *AIEE Transactions on Power Apparatus and Systems* 78, Part 3B: 1747-1752. 1959.
29. Baldwin, C. J., DeSalvo, C. A. and Limmer, H. D. The effect of unit size, reliability, and system service quality in planning generation expansion. *AIEE Transactions on Power Apparatus and Systems* 79, Part 3: 1042-1050. 1960.
30. Fitzpatrick, R. J. and Gallagher, J. W. Determination of an optimized generation expansion pattern. *AIEE Transactions on Power Apparatus and Systems* 80, Part 3: 1052-1059. 1961.
31. Bailey, E. S., Jr., Galloway, C. D., Hawkins, E. S. and Wood, A. J. Generation planning programs for interconnected systems, Part I - Expansion programs. *IEEE Transactions on Power Apparatus and Systems, Special Supplement*: 761-774. 1963.
32. Bailey, E. S., Jr., Galloway, C. D., Hawkins, E. S. and Wood, A. J. Generation planning programs for interconnected systems, Part II - Production cost programs. *IEEE Transactions on Power Apparatus and Systems, Special Supplement*: 775-788. 1963.
33. Galloway, C. D. and Garver, L. L. Computer design of single-area generation expansions. *IEEE Transactions on Power Apparatus and Systems* 83, No. 4: 305-310. April, 1964.
34. Garver, L. L. Effective load carrying capability of generating units. *IEEE Transactions on Power Apparatus and Systems* 85, No. 8: 910-919. August, 1966.
- 34-a Dale, K. D. Dynamic programming approach to the selection and timing of generation plant additions. *IEEE Proceedings* 113: 803-811. 1966.



- 34-b Balériaux, H. Dynamic optimization of a range of generating plant in a combination of which expansion continues. *Electricite. Belgium.* 127: 19-26. July 1966.
- 34-c Bennett, R. R. Planning for power - A look at tomorrow's station sizes. *IEEE Spectrum* 5, No. 9: 67-72. September 1968.
- 34-d Ogawa, H. A new method of power-generation planning-determination of service dates. *Elect. Engng. Japan.* 89: 68-77. February, 1969.
- 34-e Ringlee, Robert J. and Wood, Allen J. Frequency and duration methods for power system reliability calculation: Part II - Demand model and capacity reserve model. *IEEE Transactions on Power Apparatus and Systems* 88, No. 4: 375-388. April 1969.
- 34-f Galloway, Charles D., Garver, Len L., Ringlee, Robert J. and Wood, Allen J. Frequency and duration methods for power system reliability calculations, Part III - Generation system planning. *IEEE Transactions* 88, No. 8: 1216-1223. August 1969.
35. Karlicek, R. F. A digital computer program for evaluating power system expansion plans. *IEEE Winter Power Meeting, Paper No. 70 CP 157-PWR.* January 1970.
36. Henault, E. C., Eastvedt, R. B., Peschon, J. and Hajdu, L. P. Power system long-term planning in the presence of uncertainty. *IEEE Transaction on Power Apparatus and Systems* 89, No. 1: 156-165. January, 1970.
37. Blatt, J. M. Introduction to FORTRAN IV programming. Pacific Palisades, California, Goodyear Publishing Company. 1968.
38. IBM system/360 operating system, Sort/Merge, IBM systems reference library, File No. S360-33, Fo-m C28-6543-5. 1968.
39. IBM system/360 operating system PL/I(F) programmer guide. IBM system reference library, File No. S360-32, Form C28-6594-4. 1968.
40. IBM system/360 operating system utilities. IBM system reference library, File No. S360-32, Form C28-6586-4. 1967.

41. Anderson, P. M. The world's energy supply. Unpublished notes. Ames, Iowa, Dept. of Electrical Engineering, Iowa State University. 1967.
42. Snedecor, G. W. and Cochran, W. G. Statistical methods. 6th ed. Ames, Iowa, Iowa State University Press. 1967.
43. Cihlar, T. C., Wear, J. H., Ewart, D. N. and Kirchmayer, L, K. Electrical utility system security. Unpublished paper presented at the American Power Conference, Chicago, Illinois, April 21, 1969. Chicago, Illinois. 1969.
44. Fry, T. C. Probability and its engineering uses. Princeton, New Jersey, D. Van Nostrand Company Inc. 1968.
45. Calabrese, G. Generating reserve capacity determined by the probability method. AIEE Transactions 66: 1439-1450. 1947.
46. Loane, E. S. and Watchorn, C. W. Probability methods applied to generator capacity problem of a combined hydro and steam system. AIEE Transactions 66: 1645-1654. 1947.
47. Watchorn, C. W. Elements of system capacity requirements. AIEE Transactions 70, Part 2: 1163-1185. 1951.
48. Calabrese, G. Determination of reserve capacity by the probability method. AIEE Transactions 69, Part 2: 1681-1689. 1950.
49. Adler, H. A. and Miller, K. W. A new approach to probability problems in electrical engineering. AIEE Transactions 65: 630-632. 1946.
50. Bellman, R. Adaptive control process - A guided tour. Princeton, New Jersey, Princeton University Press. 1961.
51. Hadely, G. Nonlinear dynamic programming. Reading, Massachusetts, Addison-Wesley Publishing Co., Inc. 1964.
52. Cotterman, T. E. and Knoop, P. A. Tables of limiting t values for probabilities to the nearest .001. Atomic Energy Report No. AMRL-TR-67-161 (Wright-Patterson Air Force Base, Ohio). May, 1968.
53. Garver, L. L. and Sager, M. A. Single area expansion program. Report No. DF-66-AD-48. User manual E-LL8. Schenectady, New York, General Electric Company. January, 1966.

54. Anderson, P. M. Annual report. Affiliate research program in electrical power. Project 460-S, No. ERI-461-1. Ames, Iowa, Engineering Research Institute, Iowa State University. May, 1969.
55. Beckmann, P. Probability in communication engineering. New York, N.Y., Harcourt, Brace and World, Inc. 1967.
56. Mood, A. M. and Graybill, F. A. Introduction to the theory of statistics. New York, N.Y., McGraw-Hill Book Co., Inc. 1963.
57. Ralston, A. A first course in numerical analysis. New York, N.Y., McGraw-Hill Book Co., Inc. 1965.
58. Ralston, A. and Wilf, H. Numerical methods for digital computers. New York, N.Y., John Wiley and Sons, Inc. 1960.
59. Miller, A. L. Details of outage probability calculations. AIEE Transactions on Power Apparatus and Systems 77, Part 3: 551-557. 1958.
60. Kist, C. and Thomas, C. J. Probability calculations for system generating reserves. AIEE Transactions on Power Apparatus and Systems 77, Part 3: 515-520. 1958.
61. Halperin, H. and Adler, H. A. Determination of reserve - Generating capability. AIEE Transactions on Power Apparatus and Systems 77, Part 3: 530-544. 1958.

## XI. ACKNOWLEDGMENTS

The author wishes to express his deep appreciation to his major professor, Dr. Paul M. Anderson, for his encouragement and advice throughout this thesis.

The author wishes also to thank Mr. W. H. Weise of Iowa-Illinois Gas & Electric Company and Mr. G. F. Walkup of Iowa Power and Light Company for providing most of the data used in this project.

Special thanks are expressed to my professors and my colleagues.

To Mrs. Fetzner, who performed the difficult typing, I wish to express my sincere thanks.

The author also wishes to acknowledge the financial support rendered to him of the Engineering Research Institute and the Government of the United Arab Republic.

Last, but by no means least, to my parents for making this thesis possible, to my wife Nagwan and my daughter Dahlia for their cheerful acceptance of a part-time husband and father during the preparation of this thesis, go the most personal acknowledgments.

## XII. APPENDIX A. DATA REDUCTION PROGRAM

A flow chart is shown in Fig. A.1 for the data reduction program which converts all individual company data sets to a common format and then merges the converted data set and forms the Pool Data Set. Table A.1 shows a summary of C.P.U. times and the number of records in each data set before and after the converting process.

It will be helpful to list the six companies in the Pool according to the following order:

1. Iowa Public Service Company (IPS).
2. Iowa Electric Light and Power Company (IELP).
3. Iowa Power and Light Company (IPL).
4. Iowa Southern Utility Company (ISU).
5. Iowa-Illinois Gas and Electricity Company (IIGE).
6. Corn Belt Power Cooperative (CRNB).

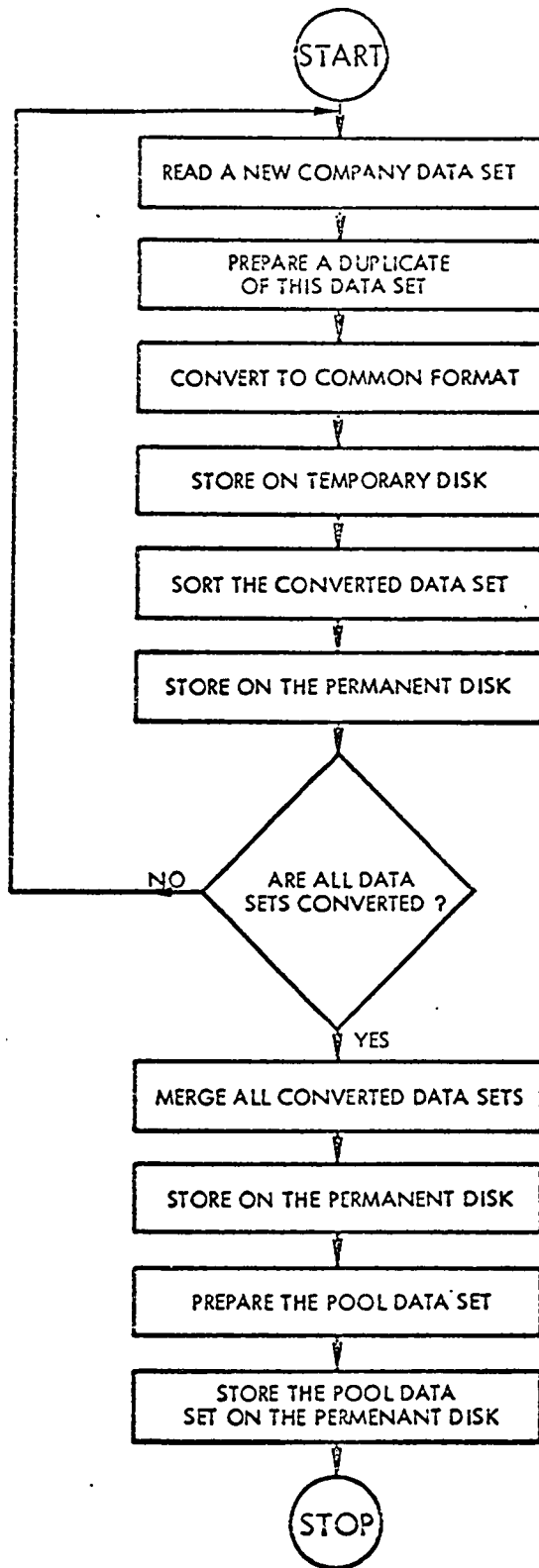
At the end of this appendix, computer listing for the six companies that convert the data to common format are shown. These listings are written in PL/I language and labeled as Program 1.

Table A.1. Summary of C.P.U. times for the Pool companies.

No.	Company	No. of records in	No. of records out	C.P.U. time for converting	C.P.U. time for sorting
1	IPS	52,584	52,584	39.00	34.00
2	IELP	96,313	96,313	51.00	52.80
3	IPL	105,204	105,143	55.00	54.90
4	ISU	6,207	74,460	22.00	32.50
5	IIGE	7,292	87,360	27.00	39.10
6	CRNB	4,384	52,608	19.00	23.80
Total No. of records			468,468		

The C.P.U. time for merging all data sets stored on disc and store the merged data set on the same disc was 107.4 seconds.

Fig. A.1. Data reduction program





```

C      *****
      /* CONVERT POWER COMPANY DATA TO STANDARD FORM FOR IPS*/
C      *****
      (SUBRG,STRG):
          IPS: PROC OPTIONS(MAIN);
          DCL DAYS(7) CHAR(3) STATIC INIT('SUN','MON','TUE','WED','THU','FRI',
            'SAT') ;
          DCL DAYNUM(7) CHAR(2) STATIC INIT('01','02','03','04','05',
            '06','07') ;
          DCL CARD CHAR(80) STATIC,
            ((MO POS(1),DAY POS(3)) CHAR(2), YR CHAR(1) POS(5),DOW
            CHAR(3) POS(6), POWER CHAR(3) POS(18), HR CHAR(2)
            POS(61)) DEFINED CARD,
            OUT CHAR(20) STATIC INITIAL ((20)'0'),
            ((MOO POS(3), DAYO POS(5),HRO POS(7),DOWO POS(9),
            COMP POS(11)) CHAR(2),YRO CHAR(1) POS(2),SIX CHAR(1) POS(1),
            POWERO CHAR(3) POS(14)) DEFINED OUT,
            (NIN INIT(0) , NOUT INIT(0)) FIXED BIN STATIC ;
          ON ENDFILE(OLD) GO TO QUIT;
          COMP = '01' ;
          SIX='6';
          NXTDAY: READ FILE(OLD) INTO(CARD); NIN= NIN+1;
                  DO I=1 TO 7 BY 1 ;
          IF DOW = DAYS(I) THEN GO TO CONT ; END ;
          PUT EDIT('DID NOT FIND DAY OF WEEK.',DOW)
            (COL(1),(2)A) ;
          I=7 ;
          CONT : DOWO = DAYNUM(I) ;
                  YRO=YR; MOO=MO;
                  DAYO = DAY ; HRO = HR ;
                  POWERO = POWER ;
                  WRITE FILE(NEW) FROM(OUT) ; NOUT = NOUT+1;
          GO TO NXTDAY ;
          QUIT: PUT EDIT(NIN,'RECORDS INPUT.',NOUT,'RECORD OUTPUT.')
            (SKIP,F(9),A,F(9),A); END;
C      *****
Program 1. Data reduction program

```

```

/*CONVERT POWER COMPANY DATA TO STANDARD FORM FOR IELP*/
C *****
(SUBRG,STRG):
  IELP: PROC OPTIONS (MAIN);
        DCL CARD CHAR(18) STATIC,
          ((YR POS(5),MO POS(1), DAY POS(3),HR POS(12)) CHAR(2) ,
           DOW POS(7) CHAR(1), POWER POS(15) CHAR(3)) DEFINED
        CARD,OUT CHAR(20) STATIC INITIAL((20)'0'),
          ((YRO POS(1),MOO POS(3),DAYO POS(5),HRO POS(7),
        COMP POS(11)) CHAR(2),DOWO POS(10) CHAR(1),POWERO CHAR(3) POS(14))
        DEFINED OUT,
          HOUR CHAR(2) INITIAL(00),HHH CHAR(9),HH CHAR(4) DEFINED HHH
          POS(6),(NIN INIT(0),NOUT INIT(0))  FIXED BIN STATIC;
        DCL BLANK CHAR(1) DEFINED OUT POS(7);
          ON ENDFILE(OLD) GO TO QUIT;
          COMP = '02';
          NXTDAY:  READ FILE(OLD) INTO(CARD); NIN=NIN+1;
                  POWERO=POWER;
                  IF HOUR = HR      THEN GO TO NXTDAY;
        YRO=YR; MOO=MO; DAYO=DAY; DOWO=DOW;HRO=HR;
          HOUR = HR ;
          IF BLANK='0' THEN BLANK=' ';
          WRITE FILE(NEW) FROM(OUT) ; NOUT=NOUT+1;
          GO TO NXTDAY;
        QUIT: PUT EDIT(NIN,'RECORDS INPUT.',NOUT,'RECORDSOUTPUT. ')
              (SKIP,F(5),A,F(6),A); END;
C *****
/* CONVERT POWER COMPANY DATA TO STANDARD FORM FOR IPL*/
C *****
(SUBRG,STRG):
  IPL: PROC OPTIONS (MAIN) ;
        DCL CARD CHAR(80) STATIC,
          ((MO POS(1),DAY POS(3),YR POS(5),HR POS(8)) CHAR(2),
           DOW POS(7) CHAR(1),POWER POS(40) CHAR(3)) DEFINED CARD ,
          ((MOX POS(1),DAYX POS(3),YRX POS(5),HRX POS(8)) CHAR(2),
           DOWX POS(7) CHAR(1),POWERX POS(77) CHAR(3)) DEFINED CARD;

```

Program 1. (Cont.)

```

      DCL OUT CHAR(20) STATIC INIT((20)'0'),
      ((YR POS(1),MOO POS(3),DAYO POS(5),HRO POS(7),COMP POS(11))
      CHAR(2),DOWO POS(10) CHAR(1),POWERO POS(14) CHAR(3)) DEFINED OUT;
      DCL (NIN INIT(0),NOUT INIT(0)) FIXED BIN STATIC;
DCL BLANK CHAR(1) DEFINED OUT POS(7);
ON ERROR GO TO QUIT;
  ON ENDFILE(OLD) GO TO QUIT;
  COMP='03';
NXTHR: READ FILE(OLD) INTO (CARD); NIN=NIN+1;
  IF NIN<62 THEN GO TO NXTHR;
  YR=YR; MOO=MO; DAYO=DAY; HRO=HR; DOWO=DOW; POWERO=POWER;
  IF BLANK='0' THEN BLANK=' ';
  WRITE FILE(NEW) FROM(OUT) ; NOUT=NOUT+1 ;
  IF YR>'61' THEN GO TO HRNXT;
  GO TO NXTHR;
HRNXT : READ FILE(OLD) INTO (CARD); NIN=NIN+1;
  YR=YRX; MOO=MOX; DAYO=DAYX; HRO=HRX; POWERO=POWERX;
DOWO=DOWX;
  IF BLANK='0' THEN BLANK=' ';
  WRITE FILE(NEW) FROM(OUT) ; NOUT=NOUT+1 ;
  GO TO HRNXT ;
QUIT: PUT EDIT(NIN,' RECORDS IN ',NOUT,' RECORDS OUT ')
      (SKIP,F(9),A,F(9),A);
      PUT EDIT (OUT)(COL(1),A);
      PUT EDIT (CARD)(COL(1),A);
      END IPL;
C *****
/* CONVERT POWER COMPANY DATA TO STANDARD FORM FOR ISU */
C *****
(SUBRG,STRG):
CONVERT: PROC OPTIONS (MAIN);
  DCL CARD CHAR (80) STATIC,
  ((YR POS (7),MO POS (3), DAY POS (5), DOW POS (9)) CHAR (2),
  POW (12) CHAR (4) POS (13)) DEFINED CARD,
  OUT CHAR (20) STATIC INITIAL ((20) '0'),
  ((YR POS (1), MOO POS (3), DAYO POS (5), HRO POS (7),

```

Program 1 (Cont.)

```

        DOWD POS (9), COMP POS (11)) CHAR (2), POWER CHAR (4)
        POS (13)) DEFINED OUT,
    HHH CHAR (9) , HH CHAR (2) DEFINED HHH POS(8),
    (NIN INIT (0), NOUT INIT (0)) FIXED BIN STATIC;
    ON ENDFILE(INFILE) GO TO ADDCARDS;
    COMP='04';
OPEN FILE(INFILE) TITLE('OLD') RECORD;
NXTDAY: READ FILE(INFILE) INTO (CARD); NIN=NIN+1;
    IF YR='66' & MO='10' & DAY='07' THEN POW(2)=' 060';
    IF YR='61' & MO='01' & DAY='20' THEN GOTO NXTDAY;
    YRO=YR; MOO=MO; DAYO=DAY;DOWD=DOW;
    DO I2=0 TO 12 BY 12;
        IF I2=12 THEN DO;
            READ FILE(INFILE) INTO (CARD);
            NIN=NIN+1; END ;
            DO I=1 TO 12 BY 1;
                HHH=I+I2; HRO=HH; POWER= POW(I);
                WRITE FILE (NEW) FROM (OUT); NOUT=NOUT+1; END; END;
            GOTO NXTDAY;
        ADDCARDS: CLOSE FILE(INFILE);
        ON ENDFILE(INFILE) GO TO QUIT;
        OPEN FILE(INFILE) TITLE('SYSIN') RECORD;
        GO TO NXTDAY;
    QUIT: PUT EDIT (NIN,' RECORDS INPUT,', NOUT, ' RECORDS OUTPUT.')
```

(SKIP,F(5),A,F(6),A); END;

```

C *****
/* CONVERT POWER COMPANY DATA TO STANDARD FORM FOR (IIGE)*/
C *****
(SUBRG,STRG):
    IIGE : PROC OPTIONS(MAIN);
            DCL CARD CHAR(120) STATIC,
            ((YR POS(10),MO POS(2),DAY POS(6)) CHAR(2),DOW POS(25)
            CHAR(1), POW(12) CHAR(6) POS(38)) DEFINED CARD,
            OUT CHAR(20) STATIC INITIAL((20)'0'),
            ((YRO POS(1),MOO POS(3),DAYO POS(5), HRO POS(7),
    COMP POS(11)) CHAR(2),DOWD POS(10) CHAR(1),POWER CHAR(3) POS(14))
Program 1 (Cont.)
```

```

        DEFINED OUT,
        HHH CHAR(9), HH CHAR(2) DEFINED HHH POS(8),
        (NIN INIT(0),NOUT INIT(0))  FIXED BIN STATIC ;
DCL POWER6 CHAR(6),POWER3 CHAR(3) DEFINED POWER6 POS(2) ;
ON ENDFILE(OLD) GO TO QUIT;
COMP='05' ;
NXTDAY : READ FILE(OLD) INTO(CARD); NIN=NIN+1;
        IF YR='61' & MO='10' & DAY ='05' THEN GO TO NXTDAY;
        IF YR='62' & MO='02' & DAY ='12' THEN GO TO NXTDAY;
        IF YR='62' & MO='06' & DAY ='29' THEN DAY='30';
        IF YR='63' & MO='08' & DAY ='22' THEN GO TO NXTDAY;
        IF YR='64' & MO='01' & DAY ='24' THEN GO TO NXTDAY;
YRO = YR ; MOO = MO; DAYO= DAY;
DOWO=DOW ;
DO I2=0 TO 12 BY 12;
    IF I2=12 THEN DO;
        READ FILE(OLD) INTO (CARD);
NIN = NIN +1 ; END;
    DO I=1 TO 12 BY 1;
        HHH = I+ I2 ; HRO = HH ;
        POWER6 =POW(I) ; POWER = POWER3 ;
WRITE FILE(NEW) FROM(OUT); NOUT = NOUT + 1; END; END;
    GO TO NXTDAY;
QUIT: PUT EDIT (NIN,' RECORDS INPUT,', NOUT, ' RECORDS OUTPUT.')
        (SKIP,F(5),A,F(6),A); END;
C *****
C /* CONVERT POWER COMPANY DATA TO STANDARD FORMAT FOR CRNBLT */
C *****
(SUBRG,STRG):
CRNB: PROC OPTIONS (MAIN);
DCL CARD CHAR(80) STATIC,
        ((MO POS(3),DAY POS(5),YR POS(7),DOW POS(9)) CHAR(2),
        POW(12) CHAR(5) POS(13)) DEFINED CARD,
        ((MOX POS(1),DAYX POS(3),YRX POS(5)) CHAR(2),POWX(12) CHAR(5)
        POS(21)) DEFINED CARD,
        OUT CHAR(20) STATIC INITIAL((20)'0'),
Program 1 (Cont.)

```

```

      ((YRO POS(1),MOO POS(3),DAYO POS(5),HRO POS(7),DOWO POS(9),
COMP POS(11)) CHAR(2), POWER CHAR(3) POS(14)) DEFINED OUT ,
      HHH CHAR(9), HH CHAR(2) DEFINED HHH POS(8),
DOWX FIXED BIN STATIC,
      (NIN INIT(0),NOUT INIT(0)) FIXED BIN STATIC ;
DCL POWER5 CHAR(5) , POWER4 CHAR(3) DEFINED POWER5 POS(3) ;
DCL BLANK2 CHAR(1) DEFINED OUT POS(5);
DCL BLANK1 CHAR(1) DEFINED OUT POS(3);
      COMP ='06';
ON ENDFILE(OLD) GO TO QUIT;
NXTDAY: READ FILE(OLD) INTO(CARD) ; NIN=NIN +1;
      DO I2= 0 TO 12 BY 12;
      IF I2 = 12 THEN DO;
READ FILE(OLD) INTO(CARD); NIN=NIN+1;
      IF YR='63' & MO='04' & DAY='16' THEN POW(11)='030';
      END ;
      DO I =1 TO 12 BY 1;
      YRO = YR ; MOO = MO ; DAYO = DAY ; DOWO = DOW ;
      HHH = I + I2 ; HRO = HH ; POWER5 = POW(I) ; POWER = POWER4 ;
IF BLANK1=' ' THEN BLANK1='0';
IF BLANK2=' ' THEN BLANK2='0';
      WRITE FILE(NEW) FROM(OUT) ; NOUT = NOUT+1;
      END; END;
      IF YR ='64' & MO ='12' & DAY ='31' THEN GO TO SECOND;
      GO TO NXTDAY;
SECOND: DOWX=DOW;
DAYNXT: READ FILE(OLD) INTO(CARD); NIN=NIN+1;
DOWX =DOWX+1 ; IF DOWX= 8 THEN DOWX=1; HHH=DOWX; DOWO=HH;
      DO I2 = 0 TO 12 BY 12;
      IF I2 = 12 THEN DO;
      READ FILE(OLD) INTO(CARD); NIN= NIN+1; END;
      DO I = 1 TO 12 BY 1;
      HHH= I +I2 ; HRO = HH ;
      YRO = YRX ; MOO = MOX ; DAYO = DAYX ;
      POWER5 = POWX(I) ; POWER = POWER4 ;
      IF BLANK1=' ' THEN BLANK1='0';
Program 1 (Cont.)

```

```
IF BLANK2=' ' THEN BLANK2='0';  
  WRITE FILE(NEW) FROM(OUT); NOUT=NOUT +1;  
END ; END ;  
  GO TO DAYNXT;  
QUIT: PUT EDIT(NIN,'RECORDS INPUT',NOUT,'RECORDS OUTPUT.')
```

(SKIP,F(5),A,F(6),A);

```
END CRNB;
```

XIII. APPENDIX B. LOAD  
MODELING PROGRAM

In this appendix, the computer flow chart for the Load Modeling Program is shown in Fig. B.1. A summary of data for January is shown in Table B.1, while a summary of the Pool energy in Mwhr is shown in Table B.2. Fig. B.2 shows the relation between the yearly energy in Mwhr's and the years. From that figure, we see that the yearly energy is increasing steadily with time. The computer listing of Program 2 is shown at the end of this appendix.



Table B.1. Summary of data for January

Year	PEAK	PBAR <sup>1</sup>	SIGMA <sup>2</sup>	PRATIO <sup>3</sup>	P.U. SIGMA <sup>4</sup>
1962	1217.0000	1142.0100	47.90121	0.93837	0.04195
1963	1324.0000	1166.81812	95.43498	0.88128	0.08179
1964	1522.0000	1322.81922	125.41904	0.86913	0.09481
1965	1413.0000	1288.69556	98.56755	0.91203	0.07649
1966	1785.0000	1560.39111	152.93324	0.87417	0.09801
1967	1893.0000	1619.86353	175.98996	0.85571	0.10864

<sup>1</sup>the mean of the monthly daily peaks.

<sup>2</sup>the standard deviation.

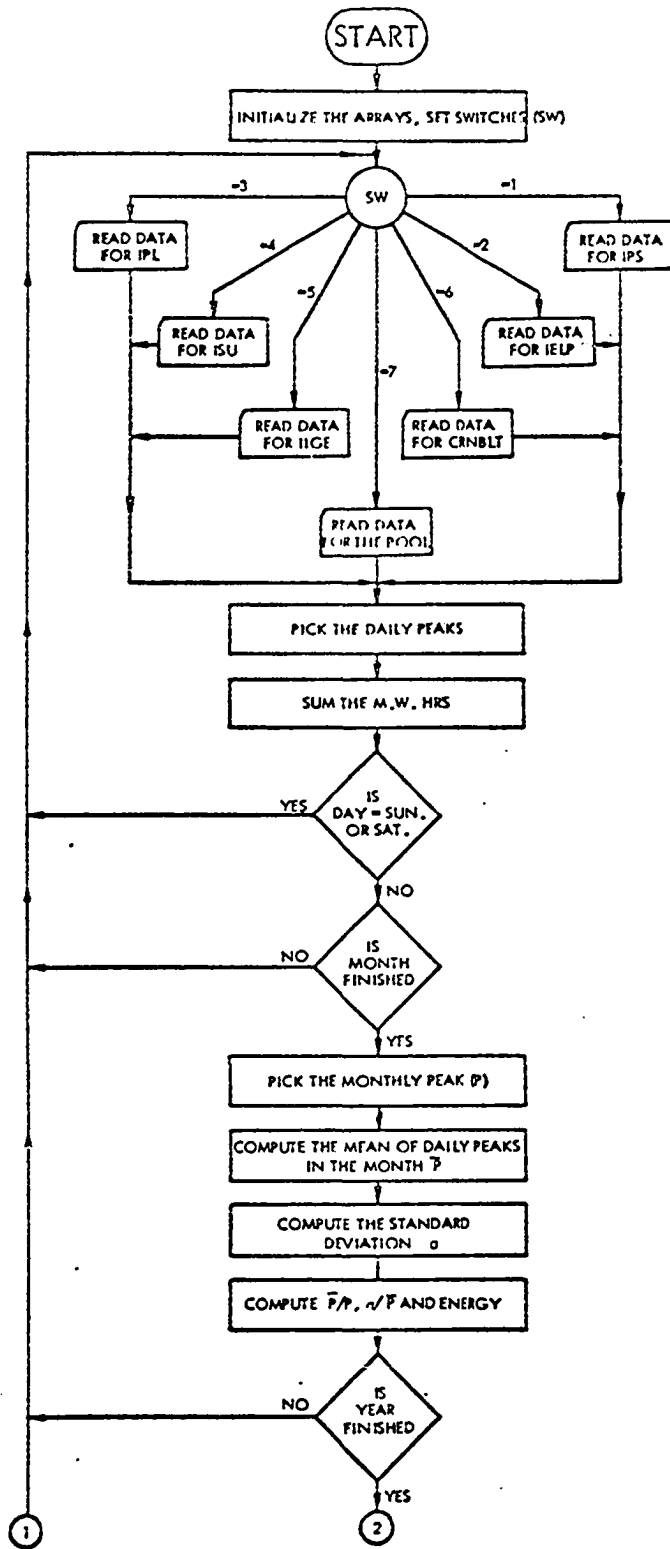
<sup>3</sup>the ratio of the mean to the peak.

<sup>4</sup>the ratio of the standard deviation to the mean.

Table B.2. Summary of energy for Pool data in Mwhr

Month	1962	1963	1964	1965	1966	1967
January	585,954	682,531	692,717	768,384	834,807	864,983
February	534,761	610,223	637,549	695,744	743,015	808,931
March	596,047	616,371	665,775	753,227	770,972	836,612
April	525,620	564,052	613,198	667,014	713,944	760,568
May	557,431	579,061	624,148	669,086	715,082	773,261
June	556,731	628,796	660,771	690,971	772,321	812,562
July	590,821	671,914	774,442	765,952	921,241	869,248
August	608,964	651,139	700,282	761,121	804,076	861,797
September	547,761	596,342	672,639	687,101	739,093	779,186
October	580,976	621,458	673,443	705,299	756,364	822,252
November	577,861	609,076	670,052	727,328	751,255	868,013
December	636,452	709,826	778,768	796,439	856,402	913,373
Yearly	<u>6,899,379</u>	<u>7,540,789</u>	<u>8,163,784</u>	<u>8,687,666</u>	<u>9,378,572</u>	<u>9,970,786</u>

Fig. B.1. Load modeling program



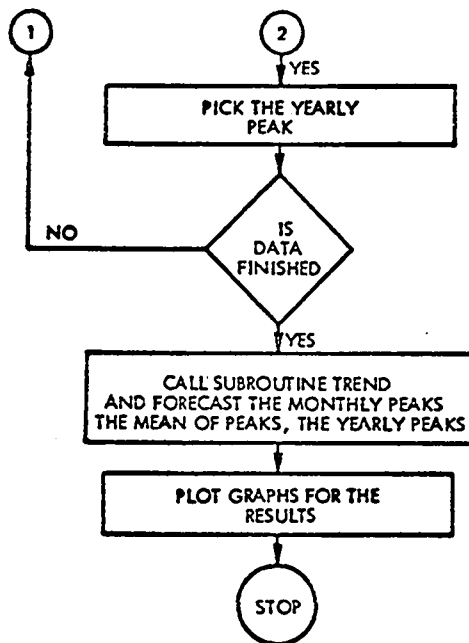


Fig. B.1 (Cont.)

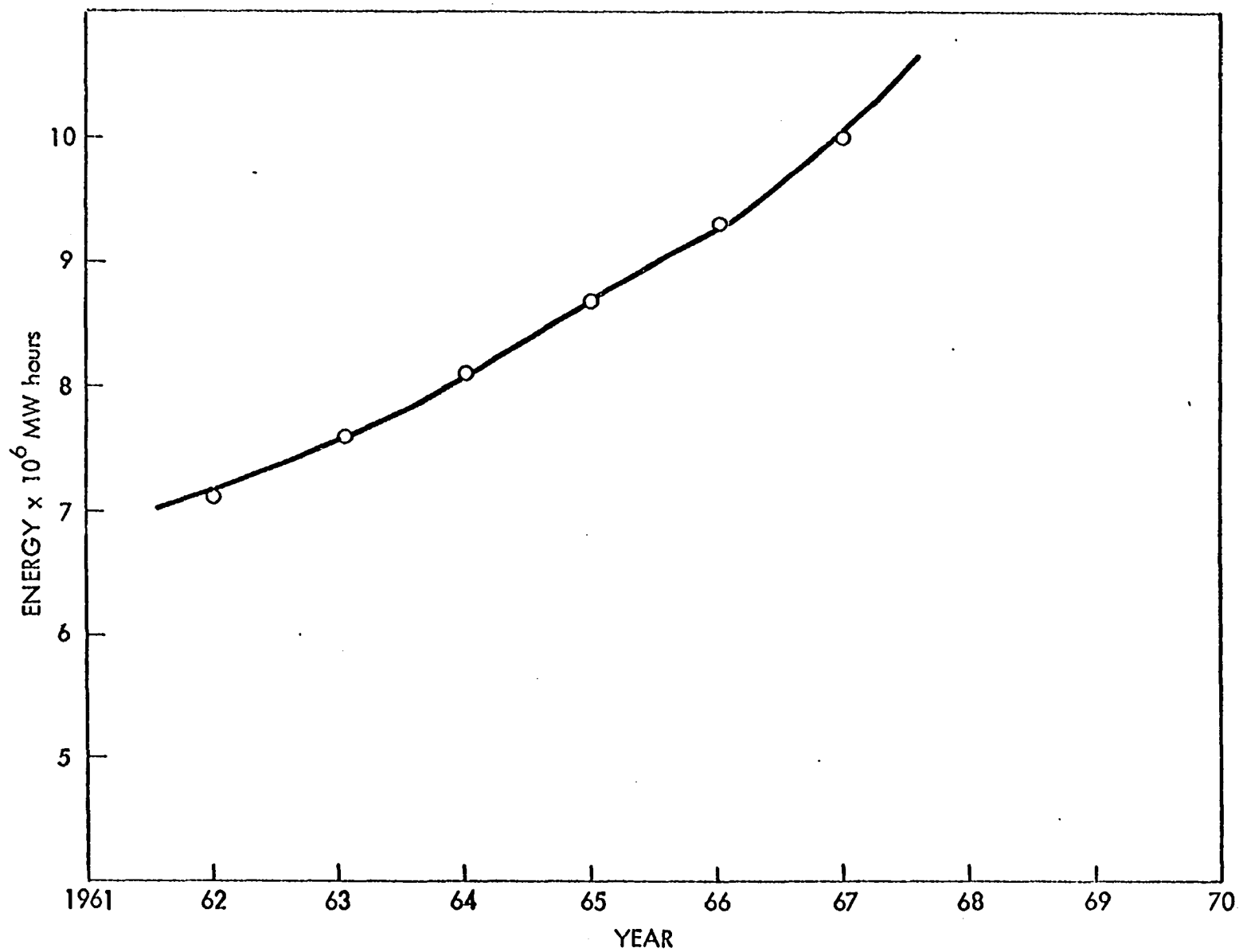


Fig. B.2. Yearly energy in MWhr versus time in years for Iowa Pool

```

C *****
C LOAD MODELING PROGRAM
C *****
  IMPLICIT LOGICAL*1(H),INTEGER*2(O),LOGICAL*4(Z)
  DIMENSION DIF(31),AY(20)
  DIMENSION HIIGE(1490),HCRNB(1490),HALL(10430),ICARD(29)
  DIMENSION IM(2),HPOOL(1490)
  DIMENSION IDAYS(7)
  DIMENSION HIPS(1490),HIELP(1490),HIPL(1490),HISU(1490)
  DIMENSION ALOAD(240),X(240)
  DIMENSION AX(20),AL(20),BL(20),CL(20),DL(20),EL(20)
  DIMENSION PAV(240),SIGMA(240),PRATIO(240),PUSIGM(240)
  DIMENSION LOAD(30),LLOAD(240),YLOAD(20)
  DIMENSION LLL(20)
  DIMENSION MO(240)
  DIMENSION ICR(29)
  DIMENSION SLOPE(400)
  DIMENSION HMON(108)
  DIMENSION INTVPK(1000)
  DIMENSION ND(240)
  DIMENSION HDUM(2),HPEAK(1170),HCOMPY(6)
  DIMENSION M(2),L(2)
  EQUIVALENCE (HALL(1),HIPS(1)),(HALL(1491),HIELP(1))
  EQUIVALENCE (HALL(2981),HIPL(1)),(HALL(4471),HISU(1))
  EQUIVALENCE (HALL(5961),HIIGE(1)),(HALL(7451),HCRNB(1))
  EQUIVALENCE (HALL(8941),HPOOL(1))
  EQUIVALENCE (IDUM,ZDUM),(HDUM(1),ODUM)
  DEFINE FILE 12(1700,373,U,IFIL12)
  DATA IDAYS/28HSUN MON TUE WED THU FRI SAT /
  DATA HMON/108HJANUARY FEBRUARY MARCH APRIL MAY JUNE
1  JULY AUGUST SEPTEMBER OCTOBER NOVEMBER DECEMBER /
  DATA HZERO/Z00/
  I2=1
  IC=7
  IIV=1
  LG=1

```

Program 2. Load modeling program

```

      IQ=0
      J=0
      LL=0
      KF=0
      NZ=0
      KZ=0
      JYX=2
      JJC=1
      KG =0
      IT =0
      IB = 55
      KHH=0
      I3 =0
      I4 =0
      MENR=0
      MENRGY=0
      KC=IC
      DO 1003 I=1,20
1003  YLOAD(I)=0.0
      AY(I)=0.0
      DO 544 LT=1,240
      PAV(LT) =0.0
      SIGMA(LT)=0.0
      PRATIO(LT)=0.0
      LLOAD(LT)=0
544  PJSIGM(LT)=0.0
700  OPEAKY=0
      K =1
      KD=1
557  GO TO(4001,4002,4003,4004,4005,4006,4166), IC
4001  DO 4449 KA=1,24
      READ(1,401,END=160) ICR(1),ICR(2),ICR(3),IDAY,ICR(5),ICR(6)
401  FORMAT(2I2,I1,A3,9X,I3,40X,I2,18X)
      IF(KA.NE.1) GO TO 4447
      ICARD(1)=ICR(1)
      ICARD(2)=ICR(2)
Program 2 (Cont.)

```



```

        ICARD(3)=ICR(3)+60
        KKC=0
        DO 1179 LV=1,7
        IF(IDAY.EQ.IDAYS(LV)) GO TO 2017
        KKC=KKC+1
1179 CONTINUE
2017 ICARD(4)=LV
4447 KP=ICR(6)+5
        ICARD(KP)=ICR(5)
4449 CONTINUE
        61 IF(KD.NE.1) GO TO 122
        M(1)=ICARD(1)
        ODUM = M(1)
        HIPS(1) = HDUM(2)
        L(1) = ICARD(3)
        ODUM = L(1)
        HIPS(1) = HDUM(2)
        OPEAKM=0
        GO TO 550
        122 M(2) = ICARD(1)
        L(2) = ICARD(3)
        IF(M(1)-M(2)) 150,550,150
4002 IIJ=0
7197 IIJ=IIJ+1
7192 READ(1,7193,END=160) (ICARD(I),I=1,7)
7193 FORMAT(3I2,11,4X,I2,1X,I3,11)
        IF(IIV.GT.2) GO TO 7198
        IIV=IIV+1
        GO TO 7192
7198 IF(IIJ.NE.1) GO TO 7194
        IM(1)=ICARD(5)
        GO TO 7196
7194 IM(2)=ICARD(5)
        IF(IM(1).EQ.IM(2)) GO TO 7192
7196 IW=5+ICARD(5)
        ICARD(IW)=ICARD(6)
Program 2 (Cont.)

```

```

        IF(IW.NE.29)  GO TO 7197
62  IF(KD.NE.1)  GO TO 222
        M(1) = ICARD(1)
        ODUM = M(1)
        HIPL(1) = HDUM(2)
        L(1) = ICARD(3)
        ODUM = L(1)
        HIPL(2) = HDUM(2)
        OPEAKM = 0
        GO TO 550
222  M(2) = ICARD(1)
        L(2) = ICARD(3)
        IF(M(1)-M(2)) 150,550,150
4003 GO TO(7099,7999),I2
7099 READ(1,7092,END=160) (ICARD(I),I=1,6)
7092 FORMAT(3I2,I1,I2,30X,I3,38X)
        IF(ICARD(3).LT.57) GO TO 4003
        IF(ICARD(3).NE.62) GO TO 7899
        I2=2
7999 READ(1,7599,END=160) (ICARD(I),I=1,6)
7599 FORMAT(3I2,I1,I2,67X,I3,1X)
7899 IQ=5+ICARD(5)
        ICARD(IQ)=ICARD(6)
        IF(IQ.NE.29) GO TO 4003
63  IF(KD.NE.1) GO TO 322
        M(1) = ICARD(1)
        ODUM = M(1)
        HIPL(1) = HDUM(2)
        L(1) = ICARD(3)
        ODUM = L(1)
        HIPL(2) = HDUM(2)
        OPEAKM = 0
        GO TO 550
322  M(2) = ICARD(1)
        L(2) = ICARD(3)
        IF(M(1)-M(2)) 150,550,150

```

Program 2 (Cont.)

```

4004 LF = 0
4014 READ(1,404,END=160) (ICARD(IJ),IJ=1,29)
404  FORMAT(2X,5I2,12I4/12X,12I4)
      IF(LF.EQ.1) GO TO 64
      IF(ICARD(3).NE.66) GO TO 1900
      IF(ICARD(1).NE.10) GO TO 1900
      IF(ICARD(2).NE.7) GO TO 1900
      ICARD(7) = 60
      LF=1
      GO TO 64
1900 IF(ICARD(3).NE.61) GO TO 64
      IF(ICARD(1).NE.1) GO TO 64
      IF(ICARD(2).NE.20) GO TO 64
      READ(1,205) (ICARD(I0),I0=1,17)
205  FORMAT(5I2,2X,12I4)
      GO TO 4014
64   IF(KD.NE.1) GO TO 422
      M(1) = ICARD(1)
      ODUM = M(1)
      HISU(1) = HDUM(2)
      L(1) = ICARD(3)
      ODUM = L(1)
      HISU(2) = HDUM(2)
      OPEAKM = 0
      GO TO 550
422  M(2) = ICARD(1)
      L(2) = ICARD(3)
      IF(M(1)-M(2)) 150,550,150
4005 READ(1,405,END=160) (ICARD(IJ),IJ=1,29)
405  FORMAT(1X,I2,2X,I2,2X,I2,13X,I1,10X,12I6,13X/35X,12I6,13X)
      IF(KF.EQ.1) GO TO 524
      IF(ICARD(3).NE.61) GO TO 501
      IF(ICARD(1).NE.10) GO TO 65
      IF(ICARD(2).NE.5) GO TO 65
      GO TO 511
501  IF(ICARD(3).NE.62) GO TO 503
Program 2 (Cont.)

```

```

IF(ICARD(1).NE.2) GO TO 502
IF(ICARD(2).NE.12) GO TO 65
GO TO 511
502 IF(ICARD(1).NE.6) GO TO 65
IF(ICARD(2).NE.29) GO TO 65
KF = 1
GO TO 65
503 IF(ICARD(3).NE.63) GO TO 504
IF(ICARD(1).NE.8) GO TO 65
IF(ICARD(2).NE.22) GO TO 65
GO TO 511
504 IF(ICARD(3).NE.64) GO TO 505
IF(ICARD(1).NE.1) GO TO 65
IF(ICARD(2).NE.26) GO TO 65
GO TO 511
505 IF(ICARD(3).NE.67) GO TO 65
IF(ICARD(1).NE.3) GO TO 65
IF(ICARD(2).NE.24) GO TO 65
511 READ(1,405) (ICARD(IJ),IJ=1,29)
GO TO 65
524 ICARD(2) = 30
65 IF(KD.NE.1) GO TO 522
M(1) = ICARD(1)
ODUM = M(1)
HIIGE(1) = HDUM(2)
L(1) = ICARD(3)
ODUM = L(1)
HIIGE(2) = HDUM(2)
OPEAKM = 0
GO TO 550
522 M(2) = ICARD(1)
L(2) = ICARD(3)
IF(M(1)-M(2)) 150,550,150
4006 GO TO(4007,4008),LG
4007 READ(1,407,END=160) (ICARD(IJ),IJ=1,29)
407 FORMAT(2X,5I2,12I5,8X/12X,12I5,8X)

```

Program 2 (Cont.)

```

        IF(ICARD(3).NE.63) GO TO 9901
        IF(ICARD(1).NE.4) GO TO 9901
        IF(ICARD(2).NE.16) GO TO 9901
        ICARD(28)=30
        GO TO 67
9901  IF(ICARD(3).NE.64) GO TO 67
        IF(ICARD(1).NE.12) GO TO 67
        IF(ICARD(2).NE.31) GO TO 67
        LG=LG+1
        GO TO 67
4008  READ(1,408,END=160) (ICARD(IJ),IJ=1,3),(ICARD(JP),JP=6,29)
408   FORMAT(3I2,14X,12I5/20X,12I5)
        ICARD(4)=ICARD(4)+1
        IF(ICARD(4).NE.8) GO TO 67
        ICARD(4)=1
67   IF(KD.NE.1) GO TO 622
        M(1) = ICARD(1)
        ODUM = M(1)
        HCRNB(1) = HDUM(2)
        L(1) = ICARD(3)
        ODUM = L(1)
        HCRNB(2) = HDUM(2)
        OPEAKM = 0
        GO TO 550
622  M(2) = ICARD(1)
        L(2) = ICARD(3)
        IF(M(1)-M(2)) 150,550,150
4166  DO 5447 KL=1,24
        READ(1,4751,END=160) (ICARD(I),I=1,6)
4751  FORMAT(5I2,6X,I4)
        KKJ=ICARD(5)+5
        ICARD(KKJ)=ICARD(6)
5447  CONTINUE
70   IF(KD.NE.1) GO TO 722
        M(1)=ICARD(1)
        L(1)=ICARD(3)
Program 2 (Cont.)

```

```

OPEAKM=0
GO TO 550
722 M(2)=ICARD(1)
L(2)=ICARD(3)
IF(M(1)-M(2)) 150,550,150
550 OPEAKD = 0
IR = ICARD(2)
4100 DO 400 JJ=6,29
II =2*(24*(IR-1)+JJ-5)+2
ODUM = ICARD(JJ)
MENR=MENR+ICARD(JJ)
GO TO(1901,1902,1903,1904,1905,1906,1907),IC
1901 HIPS(II-1)= HDUM(1)
HIPS(II) = HDUM(2)
GO TO 1911
1902 HIELP(II-1) =HDUM(1)
HIELP(II) =HDUM(2)
GO TO 1911
1903 HIPL(II-1) = HDUM(1)
HIPL(II) = HDUM(2)
GO TO 1911
1904 HISU(II-1) = HDUM(1)
HISU(II) = HDUM(2)
GO TO 1911
1905 HIIGE(II-1) = HDUM(1)
HIIGE(II) = HDUM(2)
GO TO 1911
1906 HCRNB(II-1) = HDUM(1)
HCRNB(II) = HDUM(2)
GO TO 1911
1907 HPOOL(II-1)=HDUM(1)
HPOOL(II)=HDUM(2)
1911 IN=KC
ONEWP=0
DO 800 ICOMPY=IN,KC
III=1490*(ICOMPY-1)+II

```

Program 2 (Cont.)

```

      HDUM(1) = HALL(III-1)
      HDUM(2) = HALL(III)
800  ONEWP = ONEWP+ODUM
      IF(OPEAKD.GE.ONEWP) GO TO 200
      OPEAKD = ONEWP
      ODUM = JJ-5
      HDAHR = HDUM(2)
200  IF(OPEAKM.GE.ONEWP) GO TO 300
      OPEAKM = ONEWP
      ODUM = JJ-5
      HMOHR = HDUM(2)
      ODUM = IR
      HMODA = HDUM(2)
300  IF(OPEAKY.GE.ONEWP) GO TO 400
      OPEAKY = ONEWP
      ODUM = JJ-5
      HYRHR = HDUM(2)
      ODUM = IR
      HYRDA = HDUM(2)
      ODUM = ICARD(1)
      HYRMO = HDUM(2)
400  CONTINUE
      ODUM = OPEAKD
      IF(KHH.EQ.1) GO TO 9785
      IF(ICARD(4).EQ.7) GO TO 9787
      KHH=1
      I3=I3+1
      GO TO 9787
9785 I3=I3+1
      IF(I3.NE.14) GO TO 9787
      ODUM = OPEAKM
      I4 = I4+1
      INTVPK(I4) = OPEAKM
      I3 = 0
9787 IY = ICARD(1)
      IK = 93*(IY-1) + 3*IR +54

```

Program 2 (Cont.)

```

HPEAK(IK-1) = HDUM(1)
HPEAK(IK)   = HDUM(2)
HPEAK(IK-2) = HDAHR
IF(ICARD(4).EQ.1) GO TO 417
IF(ICARD(4).EQ.7) GO TO 417
IF(ICARD(4).EQ.8) GO TO 417
LOAD(K) = OPEAKD
K=K+1
417 KD=KD+1
GO TO 557
160 L(2)= 0
150 IL = 4*M(1)+6
K = K-1
KD=KD-1
LL=LL+1
ODUM = OPEAKM
WRITE(3,9077) M(1),L(1),MENR,OPEAKM
9077 FORMAT('0',20X,I2,' - ',I2,10X,'MONTHLY HOUR LOADS = ',I10,3X,
1'M.W.HR.',10X,'MONTHLY PEAK = ',I5,3X,'M.W.')
IPEAKM=OPEAKM
LLOAD(LL)=OPEAKM
HPEAK(IL-1) = HDUM(1)
HPEAK(IL)   = HDUM(2)
HPEAK(IL-2) = HMOHR
HPEAK(IL-3) = HMODA
MENRGY=MENRGY+MENR
MENR=0
IFIL12 = 73*(L(1)-IB)+6*(M(1)-1)+IC
IF(IC.EQ.7) GO TO 1507
GO TO(1501,1502,1503,1504,1505,1506),IC
1501 WRITE(12'IFIL12) HIPS
GO TO 1507
1502 WRITE(12'IFIL12) HIELP
GO TO 1507
1503 WRITE(12'IFIL12) HIPL
GO TO 1507
Program 2 (Cont.)

```



```

1504 WRITE(12'IFIL12) HISU
      GO TO 1507
1505 WRITE(12'IFIL12) HIIGE
      GO TO 1507
1506 WRITE(12'IFIL12) HCRNB
1507 IT = IT+1
      SUM1=0.
      SUM=0.
      DO 448 MI=1,K
1448 SUM1=SUM1+FLOAT(LOAD(MI))
      PAV(IT)=SUM1/FLOAT(K)
      PRATIO(IT) = PAV(IT)/FLOAT(IPEAKM)
      ND(IT) = K
      DO 449 MM=1,K
      DIF(MM)=FLOAT(LOAD(MM))-PAV(IT)
1449 SUM = SUM + DIF(MM)*DIF(MM)
      SIGMA(IT)= SQRT(SUM/FLOAT(K-1))
      PUSIGM(IT)=SIGMA(IT)/PAV(IT)
      WRITE(3,531) M(1),PAV(IT),PRATIO(IT),SIGMA(IT),PUSIGM(IT)
531  FORMAT('0',10X,'M=',I2,10X,'M. AVG.=' ,F10.5,10X,'M. AVG. R.=' ,
      1F10.5,10X,'SIGMA=' ,F10.5,10X,'P.U. SIGMA=' ,F10.5)
      IF(IC.EQ.7) GO TO 1660
      GO TO(1601,1602,1603,1604,1605,1606),IC
1601 DO 121 I=1,1490
      121 HIPS(I) = HZERO
      GO TO 1660
1602 DO 131 I=1,1490
      131 HIPL(I) = HZERO
      GO TO 1660
1603 DO 137 I=1,1490
      137 HIPL(I) = HZERO
      GO TO 1660
1604 DO 151 I=1,1490
      151 HISU(I) = HZERO
      GO TO 1660
1605 DO 161 I=1,1490
      Program 2 (Cont.)

```

```

161 HIIGE(I)= HZERO
    GO TO 1660
1606 DO 171 I=1,1490
    171 HCRNB(I) = HZERO
1660 KD=1
    K =1
    DO 547 LT=1,30
    547 LOAD(LT)=0
        IF(L(1)-L(2)) 140,60,1477
1477 IF(M(1).NE.12) GO TO 141
    140 ODUM = L(1)
        J=J+1
        HPEAK(1) =HDUM(2)
        HPEAK(2) =HYRMD
        HPEAK(3) =HYRDA
        HPEAK(4) =HYRHR
        LIL(JJC)=L(1)
        JJC=JJC+1
        ODUM = OPEAKY
        WRITE(3,9088) L(1),MENRGY,OPEAKY
9088 FORMAT('0',20X,I3,3X,' YEARLY HOUR LOADS = ',I12,3X,'M.W.HR.',20X,
1'YEARLY PEAK=',I5,3X,'M.W.')
        MENRGY=0
        YLOAD(J)= OPEAKY
        AY(J) = J
        ODUM = OPEAKY
        HPEAK(5) = HDUM(1)
        HPEAK(6) = HDUM(2)
        IF(IC.EQ.7) GO TO 141
        WRITE(12'IFIL12) HPEAK
141 DO 110 I=1,1170
    110 HPEAK(I) = HZERO
        IF(L(2)) 666,600,666
666 OPEAKY=0
    60 GO TO (61,62,63,64,65,67,70),IC
600 JS = 1

```

Program 2 (Cont.)

```

      DO 8007 IU=1,I4
      WRITE(3,7022) IU,INTVPK(IU)
7022 FORMAT(' ',20X,' INTVPK(',I3,')= ',I5)
8007 CONTINUE
      GO TO(4110,3001),JYX
3001 JX = 0
      JKQ=(JS-1)*9+1
      JKU=JKQ+8
      WRITE(3,5017)
5017 FORMAT('1',40X,'*****
1*')
      WRITE(3,5018) (HMON(I),I=JKQ,JKU)
5018 FORMAT(' ',49X,' SUMMARY OF DATA FOR',1X,9A1)
      WRITE(3,5019)
5019 FORMAT(' ',40X,'*****
1*')
      WRITE(3,5007)
5007 FORMAT(' ', '*****
1*****
1*****')
      WRITE(3,5005)
5005 FORMAT(' ',6X,'YEAR',16X,'PEAK',17X,'PBAR',18X,'SIGMA',18X,
1'PRATIO',14X,'P.U.SIGMA')
      WRITE(3,5008)
5008 FORMAT(' ', '*****
1*****
1*****')
      IF(IC.EQ.7) GO TO 7811
      WRITE(12'IFIL12) PAV
      WRITE(12'IFIL12) SIGMA
      WRITE(12'IFIL12) PRATIO
      WRITE(12'IFIL12) PUSIGM
7811 CONTINUE
      KYX=LLL(1)+1899
      DO 588 I=JS,IT,12
      JX = JX+1
Program 2 (Cont.)

```

```

      IF(PAV(I)) 13,12,13
13  AL(JX) = PAV(I)
      BL(JX) = SIGMA(I)
      CL(JX) = PRATIO(I)
      DL(JX) = PUSIGM(I)
      EL(JX)=FLOAT(LLOAD(I))
      AX(JX)=JX
      OY=JX+KYX
      WRITE(3,1005) OY,EL(JX),AL(JX),BL(JX),CL(JX),DL(JX)
1005 FORMAT(' ',I10,5(10X,F12.5))
      588 CONTINUE
      12 JB=4*JS-3
      WRITE(3,8070)
      WRITE(3,7060) (HMON(I),I=JKQ,JKU)
7060 FORMAT(20X,' MONTHLY AVERAGE PEAK FORECASTING FOR',1X,9A1)
      WRITE(3,9800)
9800 FORMAT(20X,' *****')
      KTYPE=1
      CALL TREND(JX,AL,AX,SLOPE,JB,KYX,KTYPE)
      WRITE(3,8070)
      WRITE(3,7080) (HMON(I),I=JKQ,JKU)
7080 FORMAT(20X,' AVERAGE TO PEAK RATIO FORECASTING FOR',1X,9A1)
      WRITE(3,9800)
      JB=JB+1
      KTYPE=2
      CALL TREND(JX,CL,AX,SLOPE,JB,KYX,KTYPE)
      WRITE(3,8070)
      WRITE(3,7090) (HMON(I),I=JKQ,JKU)
7090 FORMAT(20X,' SIGMA TO AVERAGE RATIO FORECASTING FOR',1X,9A1)
      WRITE(3,9800)
      JB=JB+1
      KTYPE=3
      CALL TREND(JX,DL,AX,SLOPE,JB,KYX,KTYPE)
      WRITE(3,8070)
      WRITE(3,7050) (HMON(I),I=JKQ,JKU)
7050 FORMAT(20X,' MONTHLY PEAK FORECASTING FOR',1X,9A1)
Program 2 (Cont.)

```

```

WRITE(3,9800)
JB=JB+1
KTYPE=4
CALL TREND(JX,EL,AX,SLOPE,JB,KYX,KTYPE)
JS = JS+1
IF(JS-12) 3001,3001,3002
3002 JB=JB+1
WRITE(3,8070)
8070 FORMAT('1',19X,'*****')
WRITE(3,8050)
8050 FORMAT(20X,' YEARLY PEAK FORECASTING')
WRITE(3,9800)
N=J
KTYPE=5
CALL TREND(N,YLOAD,AY,SLOPE,JB,KYX,KTYPE)
WRITE(3,8070)
IXZ=5*JB+2
DO 5407 I=1,IXZ
WRITE(3,5022) I,SLOPE(I)
5022 FORMAT(' ',20X,' SLOPE(',I3,')=',F10.5)
5407 CONTINUE
4110 STOP
END

```

## XIV. APPENDIX C.

## LOAD FORECASTING

After organizing the raw data previously stored the daily peaks, corresponding to a certain month of the first year of the available data, are fed into a FORTRAN program (Program 2, Appendix B) to calculate the monthly peak  $P_m$ , the monthly average peak  $\bar{P}$  considering the month of 21 days, the standard deviation  $\sigma$ , and the two ratios  $\bar{P}/P_m$  and  $\sigma/\bar{P}$ . At the end, we will have the year's twelve arrays containing monthly peaks, monthly average peaks, ratios of the monthly average peak to the monthly peak and ratios of the standard deviation to the monthly average peak. These calculations are done for every company and for the Pool. These arrays are stored and used in evaluating the loss-of-load probability (LOLP) which will be discussed later.

To predict the load growth, we must choose a suitable law to fit our history data and at the same time, allow us to reliably predict future loads with a certain degree of confidence. To choose this law, we must know the relationship between the loads and the time and to find the law that fits these non-linear data. This problem is known as curve fitting or curvilinear regression. Several laws have been suggested which fit this kind of load data. We will use the law of load growth of an exponential type which gives the form

$$L = e^{mX} \tag{C.1}$$

where  $L$  is the load and  $X$  is the year that load occurs. Applying logarithms to this equation, we have

$$\log L = mX = Y \quad (\text{C.2})$$

i.e., the relation between the logarithm of the load and the corresponding year is linear with slope  $m$ . Using the least square method we can determine the value of  $m$ . Various checks insure that the chosen model fits the data points. The mathematical analysis is presented as follows:

1. From eq. (C.2) compute the mean of  $X$  and  $Y$  which will be designated  $\bar{X}$  and  $\bar{Y}$  respectively.
2. Compute the square of the deviations from the means.

Let

$$x_i = X_i - \bar{X} \quad \text{for } i = 1, 2, \dots, N \quad (\text{C.3})$$

and

$$y_i = Y_i - \bar{Y} \quad \text{for } i = 1, 2, \dots, N \quad (\text{C.4})$$

where  $N$  is the number of observations. Let

$$A = \sum_{i=1}^N x_i^2 = \sum_{i=1}^N (X_i - \bar{X})^2 = \sum_{i=1}^N X_i^2 - N\bar{X}^2 \quad (\text{C.5})$$

$$B = \sum_{i=1}^N y_i^2 = \sum_{i=1}^N (Y_i - \bar{Y})^2 = \sum_{i=1}^N Y_i^2 - N\bar{Y}^2 \quad (\text{C.6})$$

and

$$C = \sum_{i=1}^N x_i y_i = \sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^N X_i Y_i - N\bar{X}\bar{Y}. \quad (\text{C.7})$$

3. Compute the variance  $\sigma^2$  and the slope  $m$  as follows:

$$\sigma^2 = \frac{\sum_{i=1}^N x_i^2 \sum_{i=1}^N y_i^2 - \left(\sum_{i=1}^N x_i y_i\right)^2}{\sum_{i=1}^N x_i^2 (N-2)} = \frac{AB - C^2}{A(N-2)} \quad (C.8)$$

where  $(N - 2)$  is called the number of degrees of freedom in  $\sigma$ . The slope  $m$  will be given as

$$m = \frac{\sum_{i=1}^N x_i y_i}{\sum_{i=1}^N x_i^2} = \frac{C}{A} \quad (C.9)$$

4. Compute the correlation  $r$  as follows:

$$r = \frac{\sum_{i=1}^N x_i y_i}{\left(\sum_{i=1}^N x_i^2 \sum_{i=1}^N y_i^2\right)^{1/2}} \quad (C.10)$$

5. To predict the best line,

$$\hat{Y} - \bar{Y} = m(X - \bar{X}) \quad (C.11)$$

where  $\hat{Y}$  is the predicted value for  $Y$ . Now rewrite Eq. C.11 as

$$\hat{Y} = m(X - \bar{X}) + \bar{Y} \quad (C.12)$$

So, knowing the slope  $m$  and the two means  $\bar{X}$  and  $\bar{Y}$  we can calculate  $\hat{Y}$  from Eq. C.12. Applying the anti-logarithm to  $\hat{Y}$  we get the estimated value of load, namely,  $\hat{L}$ . Thus,

$$\hat{L} = e^{m(X-\bar{X})+\bar{Y}} \quad (C.13)$$

6. Prediction of load limits.



Since  $\bar{Y}$  is the mean of a sample size  $N$  from a population of mean  $\mu$  and variance  $\sigma^2$ ,  $\bar{Y}$  is only an estimate of  $\mu$  with a variance of  $\sigma^2/N$ .

Also, the sample standard deviation of the regression coefficient  $m$  is given by,

$$\sigma_m^2 = \frac{\sigma^2}{\sum x^2} = \frac{\sigma^2}{N \sum_{i=1} (X - \bar{X})^2} = \frac{\sigma^2}{A} \text{ with } N - 2 \quad (\text{C.14})$$

degrees of freedom.

Assuming the correct value of an individual member of the population as

$$Y = m_c(X - \bar{X}) + \mu + e \quad (\text{C.15})$$

where  $e$  is the error term which is a random variable drawn from a population of mean zero and variance  $\sigma^2$  and  $m_c$  is the correct value of the slope. Now the error of the prediction becomes

$$\hat{Y} - Y = (m - m_c)(X - \bar{X}) + (\bar{Y} - \mu) - e. \quad (\text{C.16})$$

The error element  $e$  for the new member is an additional source of uncertainty.

From Eq. C.16, three sources of errors exist. The first error source is due to considering  $\bar{Y}$  an unbiased estimate of  $\mu$  with a variance  $\sigma^2/N$ . The second error source is due to the error in the slope with a variance  $\sigma^2(X - \bar{X})^2/\sum(X - \bar{X})^2$ . The third error source is due to the random error term  $e$  with a variance  $\sigma^2$ . Also, the independence of the error terms guarantees that these three sources are uncorrelated, so that the variance of their sum is the sum of the three variances. Thus,

the mean square error of the predicted value will be

$$\sigma_{\hat{Y}}^2 = \frac{\sigma^2}{N} + \frac{\sigma^2 (X - \bar{X})^2}{\Sigma (X - \bar{X})^2} + \sigma^2 \quad (\text{C.17})$$

or

$$\sigma_{\hat{Y}}^2 = \sigma^2 \left[ 1 + \frac{1}{N} + \frac{(X - \bar{X})^2}{\Sigma (X - \bar{X})^2} \right] \quad (\text{C.18})$$

with  $N - 2$  degrees of freedom.

The standard error will be written as

$$\sigma_{\hat{Y}} = \sigma \left[ 1 + \frac{1}{N} + \frac{(X - \bar{X})^2}{\Sigma (X - \bar{X})^2} \right]^{1/2} . \quad (\text{C.19})$$

Using the central limit theorem, corresponding to any  $\hat{Y}$ , the point estimate of  $Y$ , there is an interval estimate

$$\hat{Y} - t_{0.05} \sigma_{\hat{Y}} \leq Y \leq \hat{Y} + t_{0.05} \sigma_{\hat{Y}} \quad (\text{C.20})$$

where  $t_{0.05}$  is the  $t$ -value at 95% confidence limits with  $N - 2$  degrees of freedom. Now substitute for  $\hat{Y}$  from Eq. C.12 in the inequality C.20. Taking the equal sign as a limiting case,

$$Y = \bar{Y} + m(X - \bar{X}) \pm t_{0.05} \sigma \left[ 1 + \frac{1}{N} + \frac{(X - \bar{X})^2}{\Sigma (X - \bar{X})^2} \right]^{1/2} \quad (\text{C.21})$$

Let

$$CI_{0.95} = \pm t_{0.05} \sigma \left[ 1 + \frac{1}{N} + \frac{(X - \bar{X})^2}{\Sigma (X - \bar{X})^2} \right]^{1/2} \quad (\text{C.22})$$

be the confidence belt at the 95% level. This is an equation of hyperbola. At  $X = \bar{X}$ , Eq. C.22 becomes

$$CI = \pm t_{0.05} \sigma \left[ 1 + \frac{1}{N} \right]^{1/2} . \quad (\text{C.23})$$

As  $X$  increases, the value  $CI$  increases indicating that the confidence interval increases as our prediction moves with time. Also, as  $X$  decreases to less than  $\bar{X}$ , the value  $CI$  increases giving the same indication about the confidence interval. Note that we have chosen the value 95% as an upper confidence limit. Now assume the value 75% as a lower confidence limit. We will have the inequality C.20 except  $t_{0.05}$  is replaced by  $t_{0.25}$ , and an equation like Eq. C.20 can be written as

$$Y = \bar{Y} + m(X - \bar{X}) \pm t_{0.25}\sigma \left[ 1 + \frac{1}{N} + \frac{(X - \bar{X})^2}{\sum (X - \bar{X})^2} \right]^{1/2} \quad (C.24)$$

Also let  $CI_{0.75}$  be defined as

$$CI_{0.75} = \pm t_{0.25}\sigma \left[ 1 + \frac{1}{N} + \frac{(X - \bar{X})^2}{\sum (X - \bar{X})^2} \right]^{1/2} \quad (C.25)$$

Since  $t_{0.25}$  is less than  $t_{0.05}$  for the same degrees of freedom, the confidence belt at the 75% limit will be contained in the 95% belt. Taking the antilogarithm for Eqs. C.21 and C.24, we can have four values of the estimated loads at the chosen confidence levels. Finally, these four estimated loads together with that given by Eq. C.13,  $\hat{L}$ , will be plotted.

7. Having the previous results, we should check the validity of the assumptions made before. These checks will be as follows (41).

1. Check the significance of the correlation coefficient  $r$ .
2. Check for a deviation that looks suspiciously large.
3. Check the homoscedacity.

These checks are done by computing a "t-value" and compared to the standard "t".

A Fortran program has been written and developed using this exponential model with all the previous checks occurring. A simplified flow chart is shown in Fig. C.1.

A "t-value" has been mentioned frequently and it is helpful to know some details about this value. Many variables have a distribution which is almost normal. The formula for the ordinate or height of a normal curve is given by

$$f = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X-\mu)^2/2\sigma^2} \quad (\text{C.26})$$

where  $X$  is the value of the variable,  $\mu$  is the population mean and  $\sigma$  is the population standard deviation. Thus two parameters,  $\mu$  and  $\sigma$ , completely determine the normal distribution. Now if we rescaled  $X$  such that its mean becomes zero and rescale  $\sigma$  to become unity, we use the transformation

$$Z = \frac{X - \mu}{\sigma} \quad (\text{C.27})$$

or

$$X = \mu + \sigma Z \quad (\text{C.28})$$

and Eq. C.26 becomes

$$f = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} \quad (\text{C.29})$$

For  $X = \bar{X}$ , the mean of a sample of size  $N$  and standard deviation equal to  $\sigma/\sqrt{N}$ , Eq. C.28 now becomes

$$\bar{X} = \mu + \frac{\sigma}{\sqrt{N}} Z. \quad (\text{C.30})$$

Since the area under the normal curve is unity, we can evaluate the probability of getting any  $Z$  by computing the area under the normal curve from 0 to  $Z$ . In most applications where the sample means are used to estimate population means and the value of  $\sigma$  is not known, we should have another distribution to enable us to compute the confidence limits for  $\mu$ , knowing  $s$  but not  $\sigma$ . This distribution is known as "student's"  $t$ -distribution. Here,  $t$  is given by

$$t = \frac{\bar{X} - \mu}{S/\sqrt{N}} \quad (\text{C.31})$$

i.e.,  $t$  is the deviation of the estimated mean from the population mean measured in  $S/\sqrt{N}$  units. Like the normal curve, the  $t$ -distribution is symmetrical about the mean but more peaked in the center and has a higher tail than the normal one. To compute  $t$  at 95% confidence limit, we should compute the  $t$  that has an area equal to 47.5%. This requires the numerical integration technique since it involves the generation of a  $\Gamma$  function. A Fortran program subroutine was used to evaluate the  $t$ -value giving the probability and vice-versa. The program uses a modified 15-point Gauss method (52).

The three following tables are typical computer outputs for January. In Table C.1, the mean or the average monthly peak is projected through 1985. The third and the fourth columns give the 95% confidence belt limits (41) and the fifth and the sixth columns give the 75% confidence belt limits. Table C.2 shows the projection for the mean to peak ratio for

the same month, while Table C.3 shows the monthly peak forecasting.

The average monthly peak forecasts for this month is plotted on Fig. C.2. The ratio of mean to peak forecasts are plotted on Fig. C.3. The monthly peak forecasts are plotted on Fig. C.4. Finally, the yearly peak forecast is shown in Fig. C.5. A list of the computer program "SUBROUTINE TREND" is given at the end of this appendix, identified as Program 3.

Table C.1. Monthly average peak forecasting for January

Year	Best estimate	95% Confidence Belt		75% Confidence Belt	
		Lower limit	Upper limit	Lower limit	Upper limit
1962	1111.8501	973.3530	1270.0522	1033.4368	1196.2119
1963	1197.3713	1059.1616	1353.6150	1119.2974	1280.8904
1964	1289.4722	1146.9634	1449.6860	1209.0664	1375.2239
1965	1388.6560	1235.1858	1561.1931	1302.0654	1481.0034
1966	1495.4702	1322.8513	1690.6123	1397.9585	1599.7820
1967	1610.4988	1409.8879	1839.6526	1496.9185	1732.6958
1968	1734.3770	1496.9170	2009.5042	1599.5181	1880.6047
1969	1867.7839	1584.8169	2201.2722	1706.4868	2044.3247
1970	2011.4502	1674.4463	2416.2783	1818.5618	2224.7957
1971	2166.1692	1766.5415	2656.1985	1936.4304	2423.1619
1972	2332.7869	1861.7090	2923.0613	2060.7219	2640.7683
1973	2512.2229	1960.4546	3219.2827	2192.0442	2879.1655
1974	2705.4583	2063.2075	3547.6301	2330.9678	3140.1108
1975	2913.5598	2170.3628	3911.2458	2478.0791	3425.5657
1976	3137.6653	2282.2712	4313.6563	2633.9597	3737.6934

Table C.1 (Cont.)

Year	Best estimate	95 Confidence Belt		75 Confidence Belt	
		Lower limit	Upper limit	Lower limit	Upper limit
1977	3379.0117	2399.2793	4758.8086	2799.2188	4078.8914
1978	3638.9224	2521.7188	5251.0781	2974.4775	4451.7930
1979	3918.8213	2649.9185	5795.3281	3160.3870	4859.2617
1980	4220.2539	2784.2190	6396.9570	3357.6404	5304.4766
1981	4544.8672	2924.9548	7061.9219	3566.9570	5790.8750
1982	4894.4531	3072.4851	7796.8359	3789.1057	6322.2461
1983	5270.9258	3227.1672	8608.9883	4024.8901	6902.7109
1984	5676.3633	3389.3782	9506.4805	4275.1719	7536.7891
1985	6112.9844	3559.5083	10498.2266	4540.8477	8229.4180



Table C.2. Average to peak ratio forecasting for January

Year	Best estimate	95% Confidence Belt		75% Confidence Belt	
		Lower limit	Upper limit	Lower limit	Upper limit
1962	0.9162	0.8483	0.9895	0.8782	0.9558
1963	0.9048	0.8428	0.9713	0.8702	0.9408
1964	0.8936	0.8350	0.9562	0.8609	0.9275
1965	0.8825	0.8247	0.9443	0.8502	0.9160
1966	0.8715	0.8118	0.9356	0.8382	0.9062
1967	0.8607	0.7970	0.9296	0.8251	0.8979
1968	0.8500	0.7806	0.9256	0.8111	0.8908
1969	0.8395	0.7634	0.9232	0.7967	0.8845
1970	0.8290	0.7456	0.9218	0.7821	0.8788
1971	0.8188	0.7277	0.9213	0.7673	0.8736
1972	0.8086	0.7097	0.9213	0.7526	0.8687
1973	0.7986	0.6918	0.9217	0.7380	0.8641
1974	0.7886	0.6742	0.9225	0.7235	0.8596
1975	0.7789	0.6569	0.9235	0.7092	0.8553
1976	0.7692	0.6398	0.9247	0.6951	0.8511

Table C.2 (Cont.)

Year	Best estimate	95% Confidence Belt		75% Confidence Belt	
		Lower limit	Upper limit	Lower limit	Upper limit
1977	0.7596	0.6231	0.9260	0.6813	0.8470
1978	0.7502	0.6068	0.9275	0.6676	0.8430
1979	0.7409	0.5908	0.9290	0.6542	0.8390
1980	0.7317	0.5752	0.9307	0.6411	0.8352
1981	0.7226	0.5600	0.9324	0.6281	0.8313
1982	0.7136	0.5452	0.9342	0.6154	0.8275
1983	0.7048	0.5307	0.9360	0.6030	0.8238
1984	0.6960	0.5165	0.9379	0.5908	0.8201
1985	0.6874	0.5028	0.9398	0.5788	0.8164

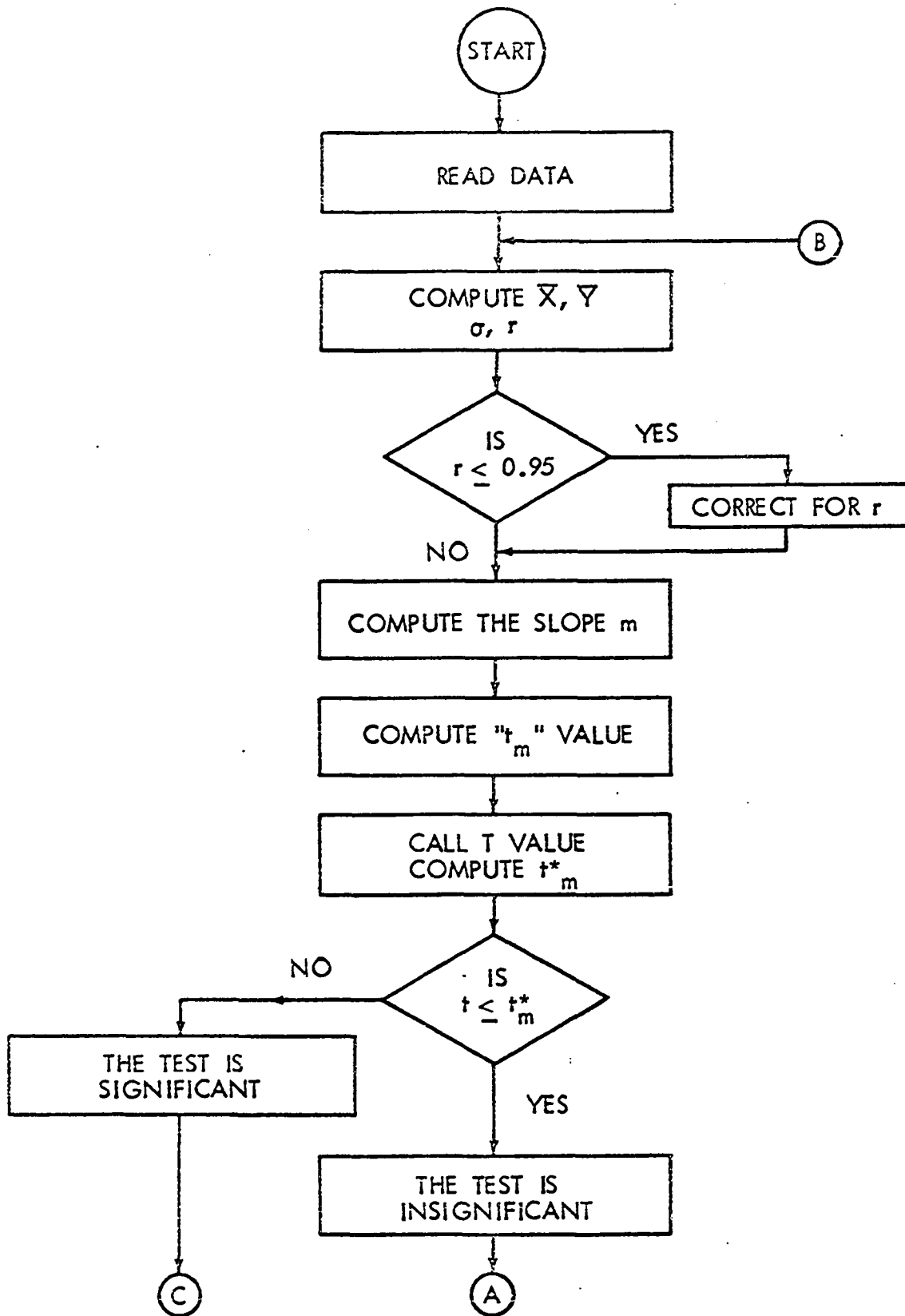
Table C.3. Monthly peak forecasting for January

Year	Best estimate	95% Confidence Belt		75% Confidence Belt	
		Lower limit	Upper limit	Lower limit	Upper limit
1962	1213.5718	1020.0515	1443.8044	1103.0376	1335.1809
1963	1323.3459	1127.5039	1553.2036	1211.8115	1445.1448
1964	1443.0500	1238.4128	1681.5000	1326.6882	1569.6160
1965	1573.5818	1350.4341	1833.6011	1446.6946	1711.5964
1966	1715.9211	1461.9817	2013.9666	1571.2996	1873.8518
1967	1871.1357	1572.7583	2226.1179	1700.7095	2058.6379
1968	2040.3901	1683.4978	2472.9395	1835.7288	2267.8667
1969	2224.9548	1795.3755	2757.3167	1977.4387	2503.4502
1970	2426.2144	1909.5786	3082.6226	2126.9617	2767.5674
1971	2645.6787	2027.1431	3452.9431	2285.3728	3062.7869
1972	2884.9951	2148.9409	3873.1584	2453.6951	3392.1042
1973	3145.9590	2275.7075	4349.0000	2632.9226	3758.9626
1974	3430.5283	2408.0818	4887.0898	2824.0347	4167.2734
1975	3740.8386	2546.6428	5495.0234	3028.0283	4621.4414
1976	4079.2183	2691.9309	6181.4453	3245.9338	5126.4141

Table C.3 (Cont.)

Year	Best estimate	95% Confidence Belt		75% Confidence Belt	
		Lower limit	Upper limit	Lower limit	Upper limit
1977	4448.2070	2844.4636	6956.1523	3478.8203	5687.7109
1978	4850.5703	3004.7593	7830.2500	3727.8113	6311.4844
1979	5289.3320	3173.3284	8816.2969	3994.1016	7004.5820
1980	4767.7813	3350.6958	9928.4648	4278.9492	7774.6367
1981	6289.5078	3537.3970	11182.7656	4583.6992	8630.1211
1982	6858.4297	3733.9880	12597.2578	4909.7813	9580.4727
1983	7478.8125	3941.0461	14192.3203	5258.7266	10636.1602
1984	8155.3125	4159.1641	15990.9648	5632.1563	11808.8125
1985	8893.0078	4388.9844	18019.0859	6031.8203	13111.3750

Fig. C.1. Subroutine trend



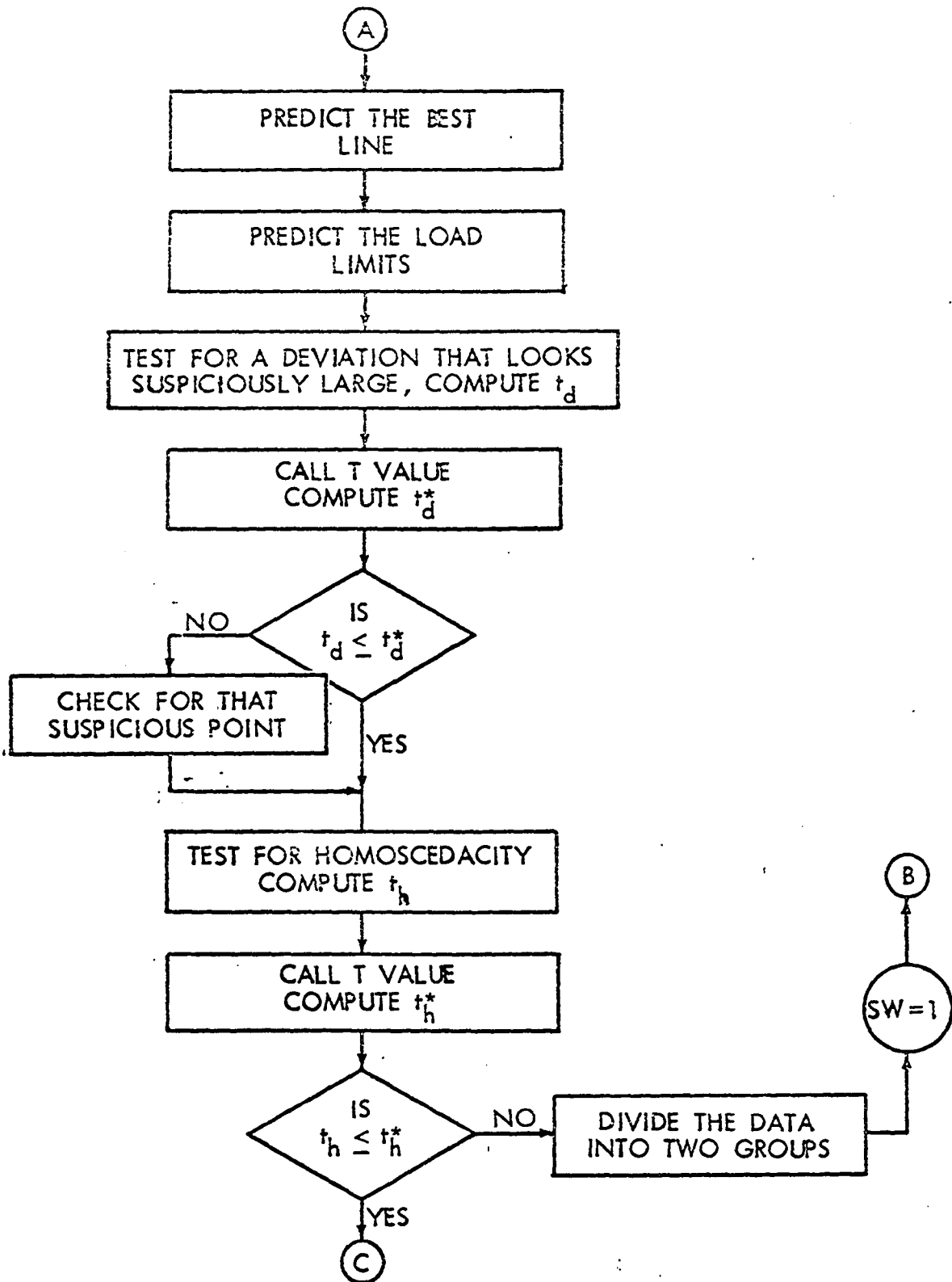


Fig. C.1 (Cont.)

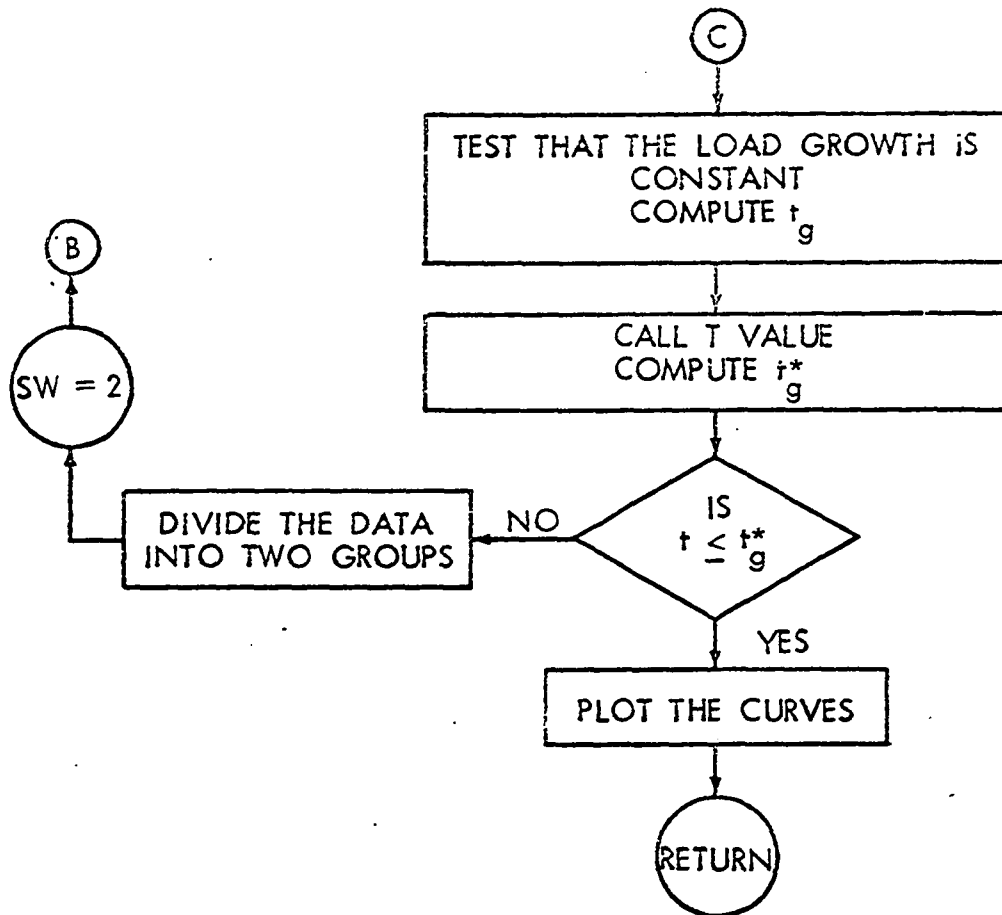


Fig. C.1 (Cont.)



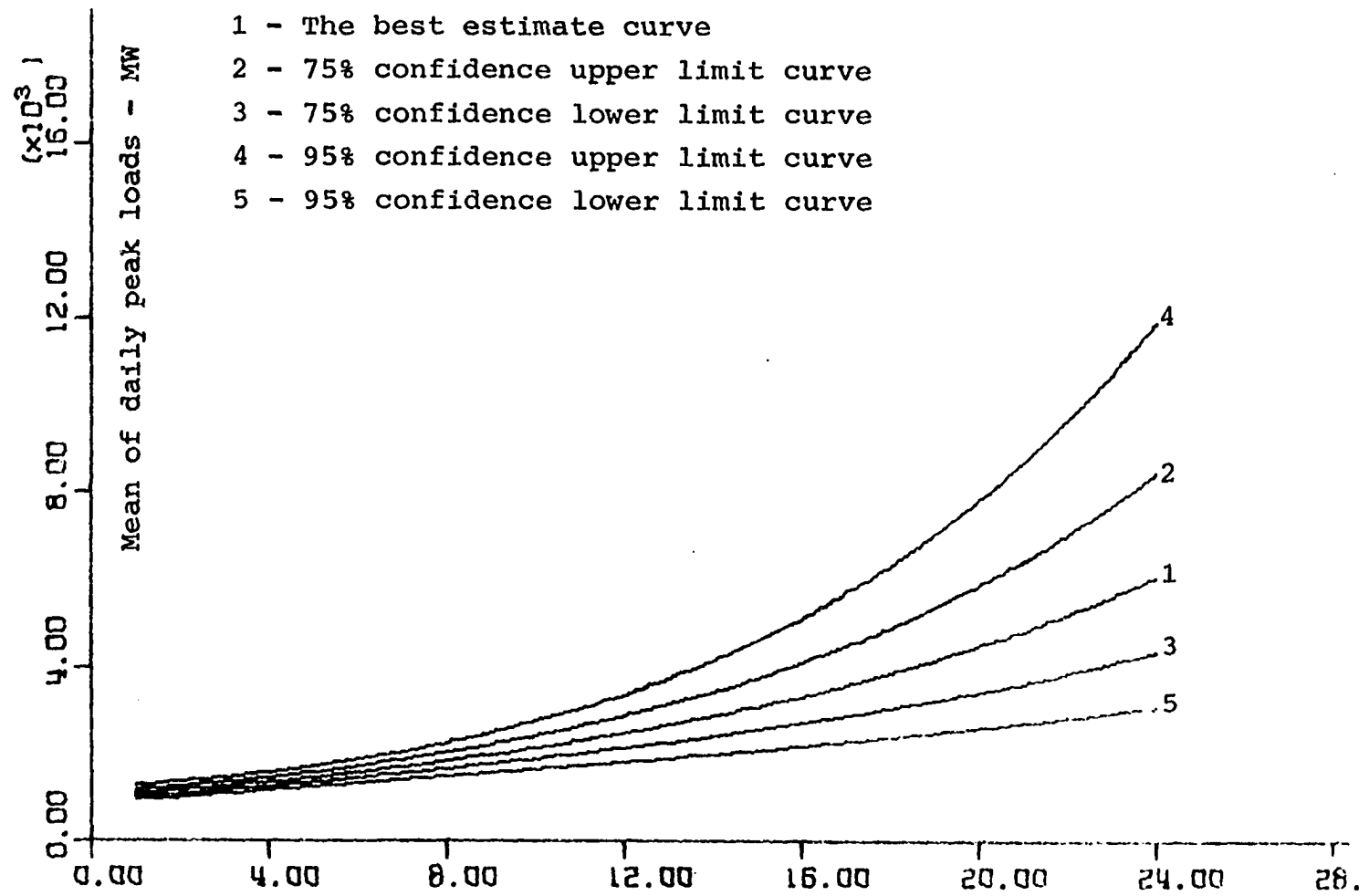


Fig. C.2. The mean of daily peak loads forecasting for January

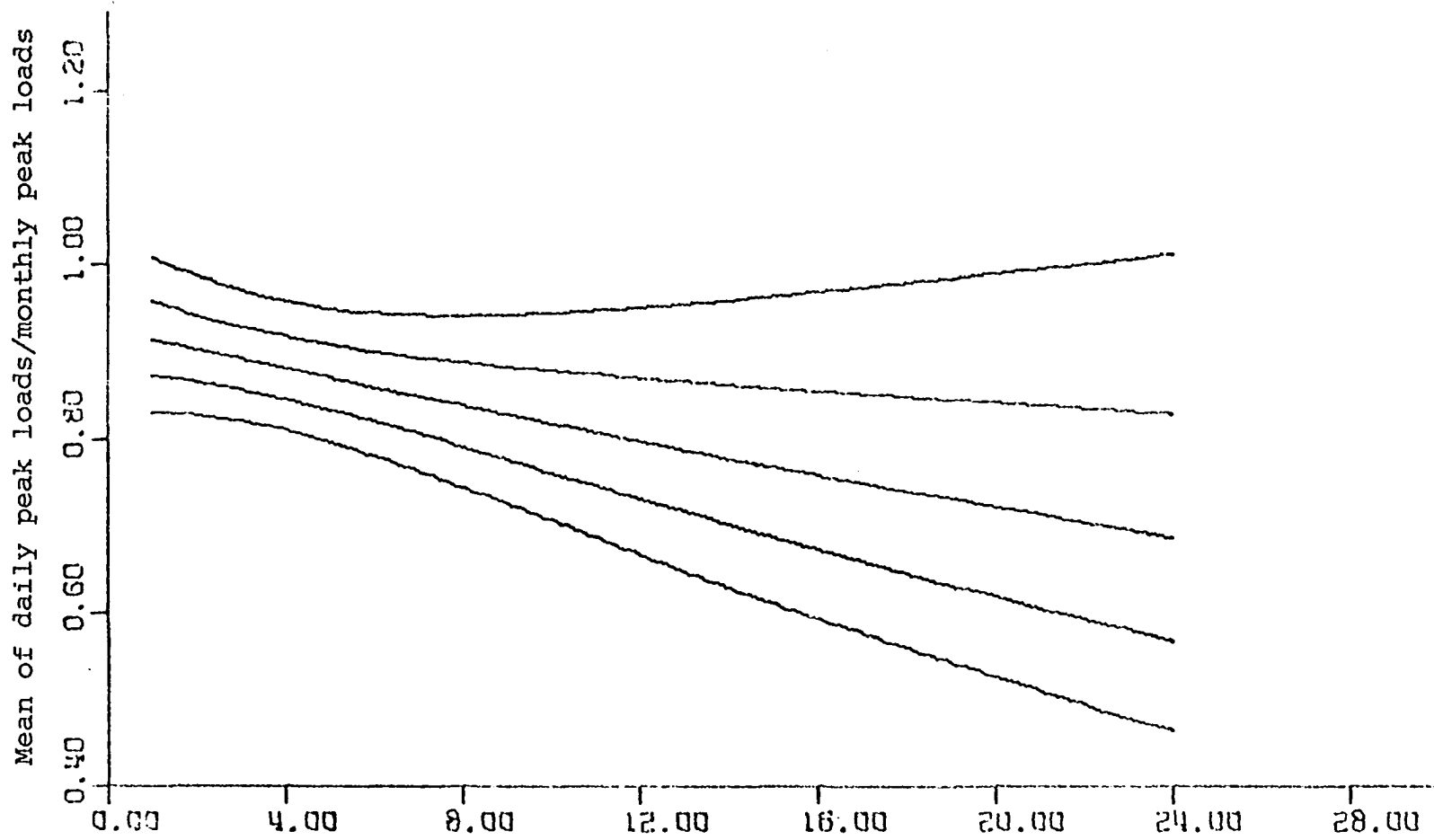


Fig. C.3. The mean of daily peaks to the monthly peaks/ratio forecast for January

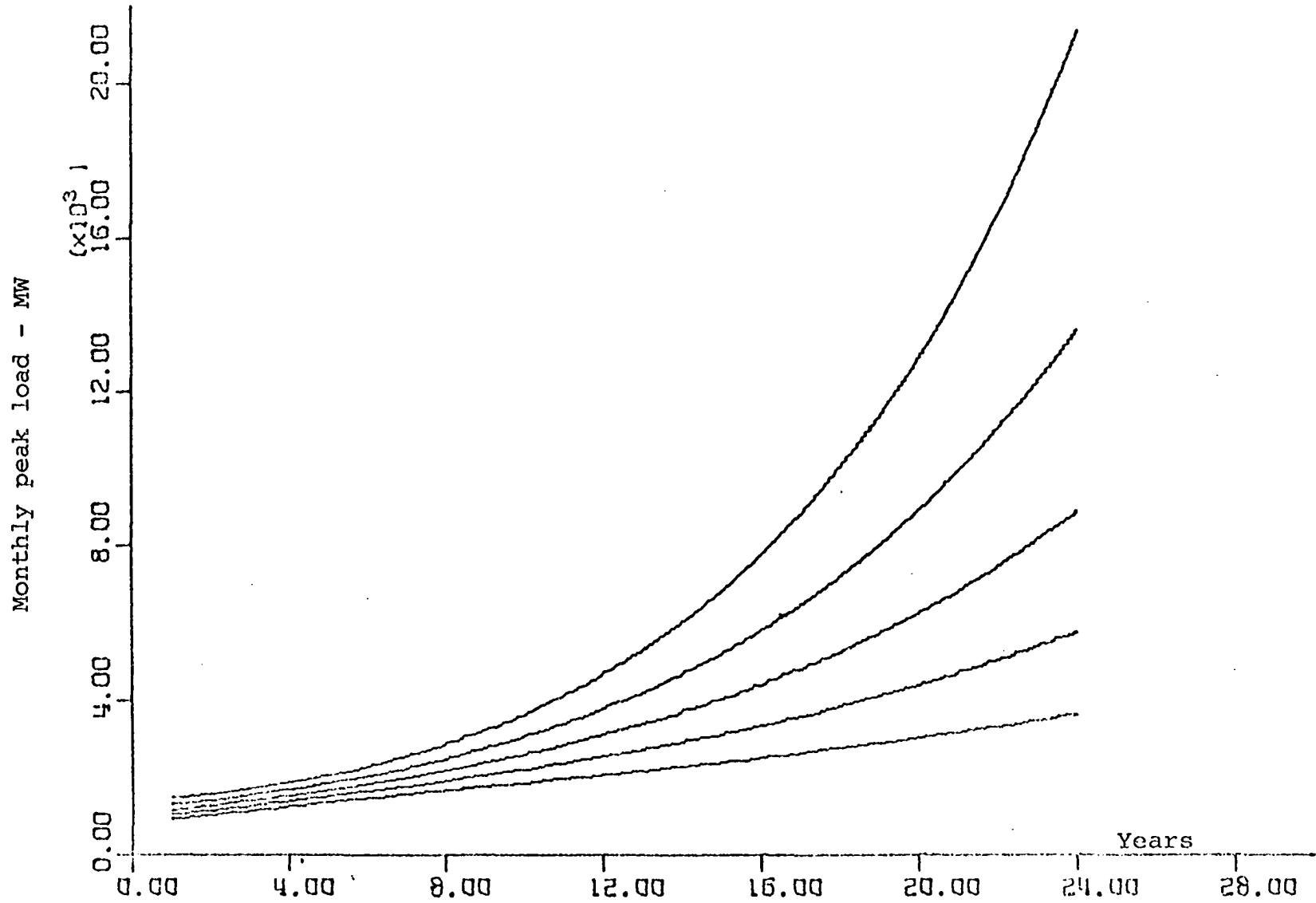


Fig. C.4. Monthly peak loads forecast for January

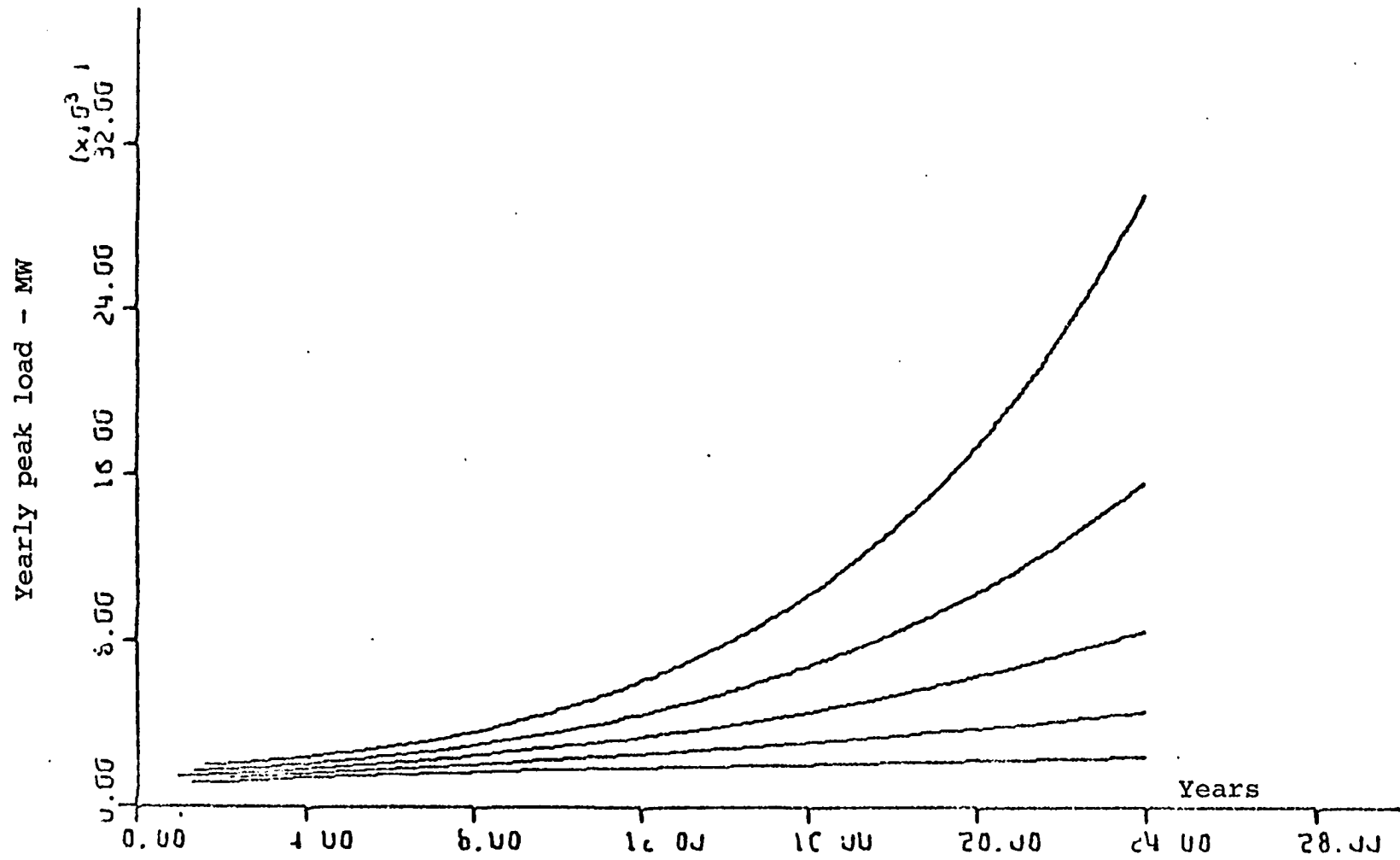


Fig. C.5. Yearly peak loads forecast

```

C *****
SUBROUTINE TREND(N,ALOAD,X,SLOPE,JB,KYX,KTYPE)
C *****
DIMENSION H1(100),H2(100),G1(100),G2(100),IYER(100)
DIMENSION SLOPE(400)
DIMENSION SUM(20),AVRG(10),DIFDB(100)
DIMENSION A(100),B(100),IT(100)
DIMENSION AK(15,50),COMPL(100),D(100),E(100),F(2,100),CC(2,100)
DIMENSION DD(2,100),G(2,100),H(2,100)
DIMENSION ALOAD(240),X(240),W(100),DIFX(100),DIFL(100),AW(100)
DIMENSION ANS(501),PP(501)
LLL=0
PMIN=0.125
PMAX=0.025
AMIN=2.75713
AMAX=1.34102
ITAPE=1
JTAPE=3
J=1
JN=N
AN=N
MK =0
SLOPE(1) = AMIN
SLOPE(2) = AMAX
5000 IF(MK-1) 20,21,22
21 J=1
N=LJ
NN=LJ
GO TO 20
22 J=NN
N=JN
20 AN=N-J+1
SUM1=0.0
SUM2=0.0
SUM3=0.0
SUM4=0.0

```

Program 3. The trend program

```

SUM5=0.0
DO 100 I=J,N
SUM1=SUM1+X(I)
W(I)=ALOG(ALOAD(I))
100 SUM2=SUM2+W(I)
AVRX=SUM1/AN
AVRL=SUM2/AN
WRITE(JTAPE,30) AVRX,AVRL
30 FORMAT('0',20X,'AVERAGE YEAR=',F10.4,20X,'AVERAGE LOAD=',F10.4)
DO 200 I=J,N
DIFX(I) = X(I)- AVRX
DIFL(I) = W(I)- AVRL
SUM3=SUM3 + DIFX(I)*DIFX(I)
SUM4=SUM4 + DIFL(I)*DIFL(I)
200 SUM5=SUM5 + DIFX(I)*DIFL(I)
SIGMA2= SQRT((SUM3*SUM4-SUM5*SUM5)/(SUM3*(AN-2.0)))
CORR = SUM5/(SQRT(SUM3*SUM4))
SLOPP=SUM5/SUM3
WRITE(3,50) SIGMA2,SLOPP,CORR
50 FORMAT('0',20X,'SIGMA = ',F10.4,20X,'SLOPE = ',F10.4,20X,
1 'CORR. COEF. = ',F10.4)
C *****
C TEST OF SIGNIFICANCE
C *****
T=AN-2.0
R= 1.0-CORR*CORR
S=CORR*SQRT(T/R)
WRITE(JTAPE,7007) S
7007 FORMAT('0',20X,' T VALUE = ',F10.5)
C *****
C PREDICTION OF BEST LINE
C *****
NX=24
DO 300 I=J,NX
X(I)=I
COMPL(I) = SLOPP*(X(I)-AVRX) + AVRL

```

Program 3 (Cont.)

```

300 D(I) = EXP(COPL(I))
IQ=5*JB-2
SLOPE(IQ) = SLOPP
SLOPE(IQ+1) = AVRX
SLOPE(IQ+2) = AVRLL
SLOPE(IQ+3) = 1.0/SUM3
SLOPE(IQ+4) = SIGMA2
IQ=IQ+4
WRITE(2,5107) (SLOPE(IJ),IJ=IQ,IQ0)
5107 FORMAT(5F12.6)
DN=D(N)
IF(LLL-1) 5001,5002,5002
IF(S-5.) 606,1015,1015
606 CALL TVALUE(N,S,AMAX,AMIN,ANS,PP)
*****
5001 CONTINUE
C
C PREDICTION OF LOAD LIMITS
*****
DO 4000 I=J,NX
X(I) = I
CV = X(I) -AVRX
E(I) = SQR(1.+1./AN+(CV*CV)/SUM3)
F(1,I) = AMIN*SIGMA2*E(I)
F(2,I) = AMAX*SIGMA2*E(I)
CO(1,I) = COPL(I)+F(1,I)
CO(2,I) = COPL(I)+F(2,I)
DO(1,I) = COPL(I)-F(1,I)
DO(2,I) = COPL(I)-F(2,I)
G(1,I) = EXP(CO(1,I))
G(2,I) = EXP(CO(2,I))
H(1,I) = EXP(DO(1,I))
H(2,I) = EXP(DO(2,I))
4000 CONTINUE
WRITE(3,5008)
5008 FORMAT(1,*,*****
)
Program 3 (Cont.)

```

```

WRITE(3,9002)
9002 FORMAT(38X,'95% CONFIDENCE BELT',22X,'75% CONFIDENCE BELT')
WRITE(3,5008)
WRITE(3,9003)
9003 FORMAT(6X,'YEAR',7X,'BEST ESTIMATE',4X,'LOWER LIMIT',10X,
1'UPPER LIMIT ',8X,'LOWER LIMIT',9X,'UPPER LIMIT')
WRITE(3,5008)
DO 400 I=J,NX
KX=I+KYX
WRITE(3,116) KX,D(I),H(1,I),G(1,I),H(2,I),G(2,I)
116 FORMAT(' ',I10,6X,F10.4,8X,F10.4,10X,F10.4,10X,F10.4,9X,F10.4)
400 CONTINUE
DO 7433 ITU=1,NX
H1(ITU)=H(1,ITU)
G1(ITU)=G(1,ITU)
H2(ITU)=H(2,ITU)
G2(ITU)=G(2,ITU)
7433 IYER(ITU)=1961+ITU
G11=G(1,N)
G22=G(2,N)
H11=H(1,N)
H22=H(2,N)
IF(MK-1) 7001,7001,7002
7002 SHD=D(NN)-DN
SHG1=G(1,NN)-G11
SHG2=G(2,NN)-G22
SHH1=H(1,NN)-H11
SHH2=H(2,NN)-H22
DO 7000 I=NN,N
D(I)=D(I)-SHD
G(1,I)=G(1,I)-SHG1
G(2,I)=G(2,I)-SHG2
H(1,I)=H(1,I)-SHH1
7000 H(2,I)=H(2,I)-SHH2
7001 CONTINUE
IF(LLL-1) 1041,1044,1044

```

Program 3.(Cont.)



```

C      *****
C      CHECKING OF HOMOSCEDACITY
C      *****
1044 IF(MK-1) 1015,1008,1041
1015 LJ=N/2
      DO 1001 I=1,N
1001 DIFOB(I)=W(I)-COMPL(I)
      DO 1019 I=6,19
1019 SUM(I) = 0.0
      ALJ=LJ
      DO 1002 I=1,LJ
1002 SUM(6)=SUM(6)+DIFOB(I)
      AVR(1)=SUM(6)/ALJ
      NN=LJ
      ALN=N-LJ+1
      DO 1003 I=NN,N
1003 SUM(7)=SUM(7)+DIFOB(I)
      AVR(2)=SUM(7)/ALN
      WRITE(3,1248) AVR(1),AVR(2),LJ
1248 FORMAT('0',20X,'AVR(1) = ',F10.5,20X,'AVR(2) = ',F10.5,20X,I2)
      AVRGT=(AVR(1)+AVR(2))/2.0
      DO 1004 I=1,2
1004 SUM(8)=SUM(8)+(AVR(I)-AVRGT)**2
      DO 1005 I=1,N
1005 SUM(9)=SUM(9)+(DIFOB(I)-AVRGT)**2
      BETWSS=SUM(8)
      WITHSS=(SUM(9)-SUM(8))/(AN-2.0)
      FRATIO=BETWSS/WITHSS
      WRITE(3,1006) FRATIO
1006 FORMAT('0',20X,'FRATIO = ',F10.5)
      TVH=SQRT(FRATIO)
      CALL TVALUE(LJ,TVH,AMAX,AMIN,ANS,PP)
      IF(TVH-ANS(26)) 1007,1007,1041
C      *****
C      COMPARISON OF REGRESSION COEFFICIENTS
C      *****

```

Program 3 (Cont.)

```

1007 DO 1009 I=1,LJ
      SUM(10)=SUM(10)+W(I)
1009 SUM(11) = SUM(11)+X(I)
      AVR(3)=SUM(10)/ALJ
      AVR(4)=SUM(11)/ALJ
      DO 1010 I=NN,N
        SUM(12)=SUM(12)+W(I)
1010 SUM(13)=SUM(13)+ X(I)
      AVR(5)=SUM(12)/ALN
      AVR(6)=SUM(13)/ALN
      DO 1020 I=1,LJ
        DIFL(I)=W(I)-AVR(3)
        DIFX(I)=X(I)-AVR(4)
        SUM(14)=SUM(14)+DIFL(I)*DIFL(I)
        SUM(15)=SUM(15)+DIFX(I)*DIFX(I)
        SUM(16)=SUM(16)+DIFX(I)*DIFL(I)
1020 CONTINUE
      DO 1030 I=NN,N
        DIFL(I)=W(I)-AVR(5)
        DIFX(I)=X(I)-AVR(6)
        SUM(17)=SUM(17)+DIFL(I)*DIFL(I)
        SUM(18)=SUM(18)+DIFX(I)*DIFX(I)
        SUM(19)=SUM(19)+DIFX(I)*DIFL(I)
1030 CONTINUE
      AS=SUM(16)*SUM(16)/SUM(15)
      BS=SUM(19)*SUM(19)/SUM(18)
      AAN=AN-4.
      SS=(SUM(14)+SUM(17)-AS-BS)/AAN
      DS=1/SUM(15)+1/SUM(18)
      SIGMAB=SQRT(SS*DS)
C *****
C TO TEST THE SIGNIFICANT OF THE DIFFERENCE IN REGRESSION COEFFICIENTS
C *****
      FS=SUM(16)/SUM(15)
      GS=SUM(19)/SUM(18)
      TVR=(ABS(FS-GS))/SIGMAB

```

Program 3 (Cont.)

```

        WRITE(3,1143)  SS,SIGMAB,FS,GS,TVR
1143  FORMAT('0',15X,'SS=',F10.5,10X,'SIGMAB=',F10.5,10X,'FS=',F10.5,
        110X,'GS=',F10.5,10X,'TVR=',F10.5)
        MN=N-2
        CALL TVALUE(MN,TVR,AMAX,AMIN,ANS,PP)
        IF(TVR-ANS(26)) 1041,1041,1008
1008  MK=MK+1
        GO TO 5000
1041  RETURN
        END
C      *****
C      SUBROUTINE TVALUE(N,S,AMAX,AMIN,ANS,PP)
C      *****
        DIMENSION  ANS(501),PP(501)
        PMIN=0.125
        PMAX=0.025
        AMAX=0.0
        AMIN=0.0
        MMM=N
        M=MMM-2
        G=120.0
        P=.0005
        IF(M-1) 300,300,301
300  G=1300.0
301  CONTINUE
        JK=501
        I=1
        84 IF(G-4.0) 98,98,99
        99 IF(G-9.0) 60,60,20
        60 CALL STAMP(M,G,GA)
            IF(GA-P) 61,41,41
        61 G =G-1.0
            GO TO 60
        20 CALL STAMP(M,G,GA)
            IF(GA-P) 40,41,41
        40 G =G -10.0

```

Program 3 (Cont.)

```

      GO TO 20
41 IF(GA-P) 42,43,43
43 G=G+0.5
      CALL STAMP(M,G,GA)
      GO TO 41
42 IF(GA-P) 44,45,45
44 G =G -.1
98 CALL STAMP(M,G,GA)
      GO TO 42
45 IF(GA-P) 46,47,47
47 G =G +0.05
      CALL STAMP(M,G,GA)
      GO TO 45
46 IF(GA-P) 48,49,49
48 G =G-.01
      CALL STAMP(M,G,GA)
      GO TO 46
49 IF(GA-P) 50,51,51
51 G=G+0.005
      CALL STAMP(M,G,GA)
      GO TO 49
50 IF(GA-P) 52,53,53
52 G=G-0.001
      CALL STAMP(M,G,GA)
      GO TO 50
53 IF(GA-P) 54,55,55
55 G=G+0.0001
      CALL STAMP(M,G,GA)
      GO TO 53
54 IF(GA-P) 56,57,57
56 G=G-.00001
      CALL STAMP(M,G,GA)
      GO TO 54
57 IF(ABS(GA-P)-0.0001) 87,88,88
88 G=i.5*G
      GO TO 84

```

Program 3 (Cont.)

```

87 G=G+0.00001
   PP(I)=P-0.0005
   ANS(I)=G
   WRITE(3,2345) I,PP(I),I,ANS(I)
2345 FORMAT('0',20X,'PP(',I3,')=' ,F10.5,20X,'ANS(',I3,')=' ,F10.5)
   IF(I-JK) 488,487,487
488 IF(P-.13) 100,100,487
100 P=P+0.001
   I=I+1
   GO TO 84
487 AMIN=ANS(26)
   AMAX=ANS(126)
   WRITE(3,2229) AMIN,AMAX
2229 FORMAT('0',20X,' TMIN =' ,F10.8,20X,' TMAX =' ,F10.8)
   WRITE(3,2244) M
2244 FORMAT('0',20X,'DEGREES OF FREEDOM = ',I2)
   RETURN
   END
C *****
SUBROUTINE GAUSS(M,T,STU)
C *****
DIMENSION V(7),VM(7),U(7)
AM=M
V(1)=0.2011940940
V(2)=0.3941513471
V(3)=0.5709721726
V(4)=0.7244177314
V(5)=0.8482065834
V(6)=0.9372733924
V(7)=0.9879925180
XPA=-(AM+1.0)/2.0
DO 660 I=1,7
660 VM(I)=-V(I)
U(1)=0.1984314853
U(2)=0.1861610000
U(3)=0.1662692058

```

Program 3 (Cont.)

```

U(4)=0.1395706779
J(5)=0.1071592205
U(6)=0.0703660475
U(7)=0.0307532420
XT=ABS(T)
A=0.0
B=3.0
IJK=100
STU=0.0
4804 IF(XT-B) 4800,4800,4801
4800 B=XT
IJK=-100
4801 CONTINUE
SUM1=((1.0+(B+A)**2/(4.0*AM))**XPA)*(0.2025782419)
DO 661 I=1,7
PHI1=(1.0+((B-A)*V(I)+B+A)**2/(4.0*AM))**XPA
PHI2=(1.0+((B-A)*VM(I)+B+A)**2/(4.0*AM))**XPA
SUM1=SUM1+U(I)*(PHI1+PHI2)
661 CONTINUE
SUMST=SUM1*(B-A)/2.0
STU=STU+SUMST
IF(IJK) 4802,4803,4803
4803 A=A+3.0
B=B+3.0
GO TO 4804
4802 IF(T) 4805,4806,4806
4805 STU=-STU
4806 CONTINUE
RETURN
END
FUNCTION GOFM(M)
AAMM= M
SQMM=SQRT(AAMM)
PI=3.14159265
IF(M-2) 200,201,202
200 GOFM=1.0/PI

```

Program 3 (Cont.)

```

        RETURN
201  GOFM=0.5/SQMM
        RETURN
202  ANSTAR=1.0
        N=M-1
206  IF(3-N) 203,204,205
203  AN=N
        BN=N-1
        ANSTAR=ANSTAR*AN/BN
        N=N-2
        GO TO 206
204  GOFM=ANSTAR*(.75)/SQMM
        RETURN
205  GOFM=ANSTAR*2.0/(SQMM*PI)
        RETURN
        END
        SUBROUTINE STAMP(M,G,GA)
        AAM=M
        CALL GAUSS(M,G,STU)
        STU=STU*GOFM(M)
        IF(ABS(STU)-0.5) 405,405,404
404  IF(STU) 406,407,407
406  STU=-0.5
        GO TO 405
407  STU=0.50
405  STU=0.50-STU
        GA=STU
        RETURN
        END

```

Program 3 (Cont.)

## XV. APPENDIX D. LOAD

## DURATION CURVES

The load duration curve is a curve showing the total time within a specified period during which the load equaled or exceeded the value shown (41). In order to construct this curve, both loads and their respective durations should be available. The per unit load duration curves have the advantage of being suitable for load variations and they can be used for future periods of time using the correct multipliers for both the per unit loads and the per unit times. Such curves are required in evaluating the production costs of the existing units in the system as well as the new generating capacity additions. The following example will show the per unit load duration curve construction.

Assume that a load is measured using a 60-minute integrating wattmeter. The readings for one day are as shown in Table D.1. The first column will show the hour during which the load in MW was recorded. The second column will show the load in MW while the third column gives the load in per unit based on the peak load of that day.

We now sort these per unit loads in decreasing order as shown in Table D.2. The first column will show the order of the sorted per unit loads in the array, while the second column shows the sorted loads. The total time which the load equaled or exceeded 1.0 p.u. is 1 hour. In per unit, this time will be  $1/24 = 0.04166$ . The total time which the load equaled or



Table D.1. Hourly loads

Hour	Load-MW	Load in p.u.
12M-1	82.0	0.820
1-2	85.0	0.850
2-3	86.0	0.860
3-4	87.0	0.870
4-5	83.0	0.830
5-6	82.0	0.820
6-7	87.5	0.875
7-8	89.0	0.890
8-9	90.0	0.900
9-10	91.0	0.910
10-11	90.0	0.900
11-12N	89.0	0.890
12-1	92.0	0.920
1-2	87.0	0.870
2-3	88.0	0.88
3-4	89.5	0.895
4-5	91.0	0.910
5-6	95.0	0.950
6-7	100.0	1.000
7-8	97.0	0.970
8-9	94.0	0.940
9-10	90.5	0.905
10-11	86.0	0.860
11-12M	82.0	0.820

Table D.2. Load-duration in p.u.

Order in the array	Sorted load in p.u.	Duration in p.u.
1	1.000	0.04166
2	0.970	0.08333
3	0.950	0.12500
4	0.940	0.16667
5	0.920	0.20833
6	0.910	-----
7	0.910	0.29166
8	0.905	0.33333
9	0.900	-----
10	0.900	0.41666
11	0.895	0.45833
12	0.890	-----
13	0.890	0.54166
14	0.880	0.58333
15	0.875	0.6250
16	0.870	-----
17	0.870	0.70833
18	0.860	-----
19	0.860	0.79166
20	0.850	0.83333
21	0.830	0.87500
22	0.820	-----
23	0.820	-----
24	0.820	1.0000

or exceeded 0.97 p.u. is 2 hours ( $2/24 = .08333$ ). The duration in p.u. for a load equal to 0.90 p.u. is equal to  $10/24 = 0.4166$ . These durations are shown on the third column of Table D.2. The load duration curve for the given day is shown in Fig. D.1.

If we approximate the relation between the per unit loads and their respective durations with a straight line relationship using the least square method (10), we can write the equation of that straight line as

$$y = mx + C \quad (D.1)$$

where  $y$  is the load in p.u.  $m$  is the slope of this line and  $C$  is the intercept of this line with the load axis. This is shown in Fig. D.2.

The area under this curve gives the energy of that day in p.u. and we have to store only  $y_1$  and  $y_2$  to define the load duration curve.

This was done for the twelve months of the Pool data and the monthly load duration curves are shown in Fig. D.3 to Fig. D.14. Fig. D.3 shows the exact curve and the straight line approximation for that curve. Fig. D.15 shows the p.u. monthly peak for 1967 of Pool data based on the 1967 peak.

A flow chart is shown in Fig. D.16 for SUBROUTINE LOADUR while a Fortran list is shown at the end of this appendix for the same subroutine, identified as Program 4.

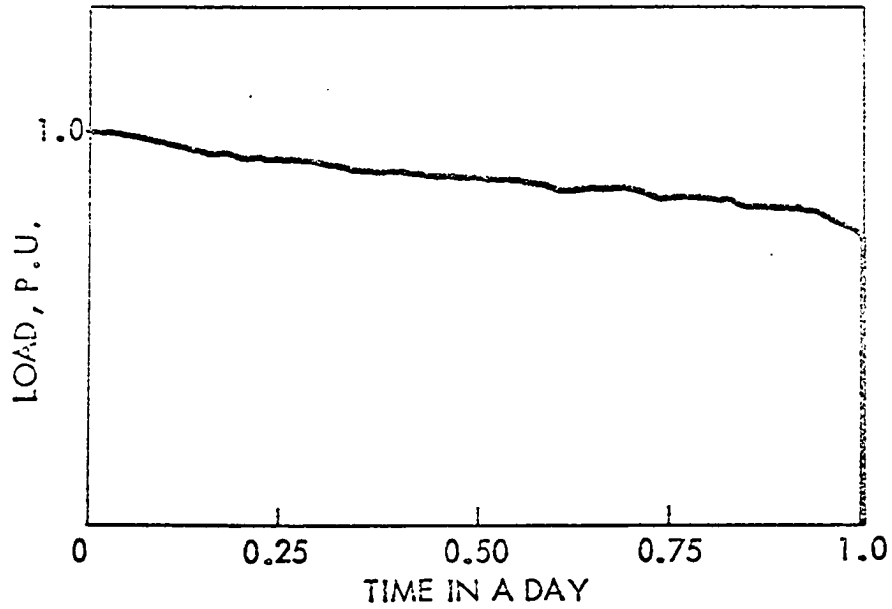


Fig. D.1. Daily load duration curve

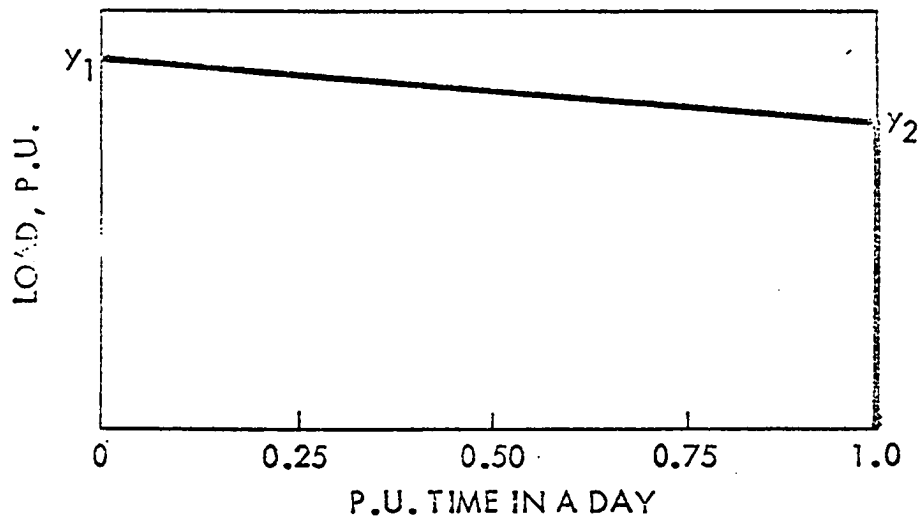


Fig. D.2. Daily load duration curve using a straight line approximation

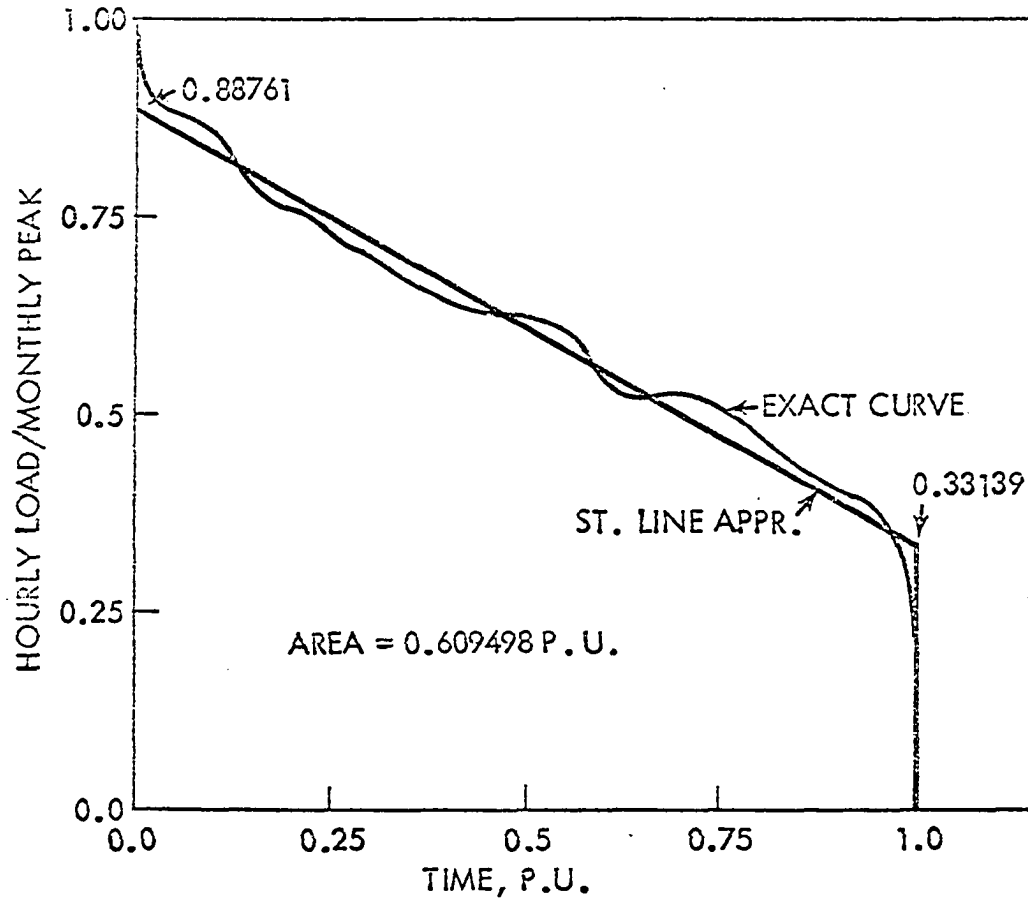


Fig. D.3. Monthly load duration curve for January

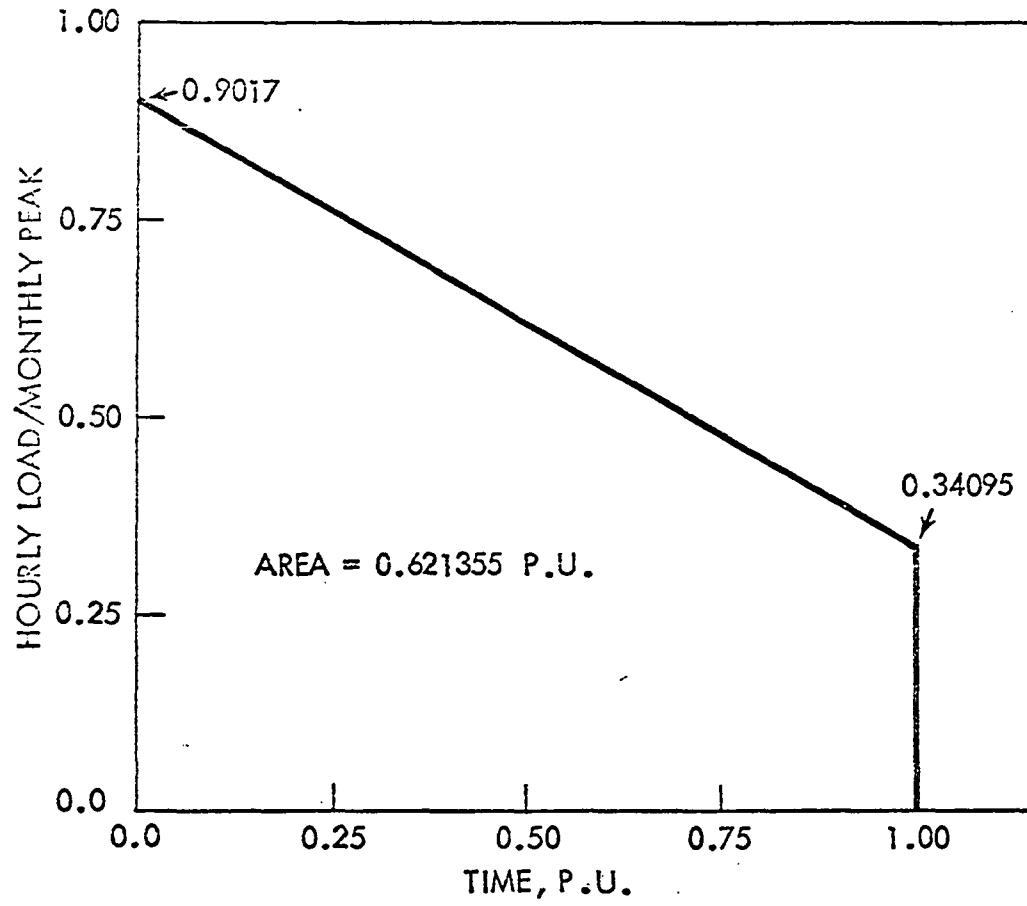


Fig. D.4. Monthly load duration curve for February

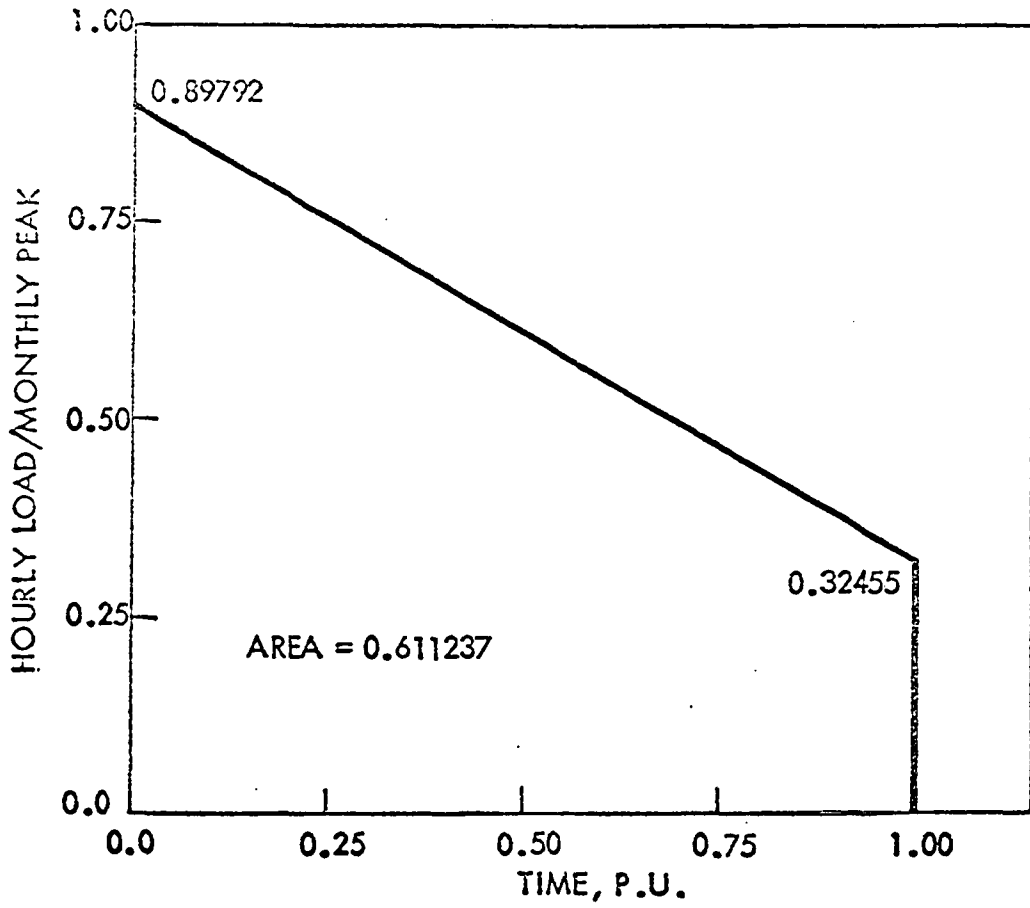


Fig. D.5. Monthly load duration curve for March

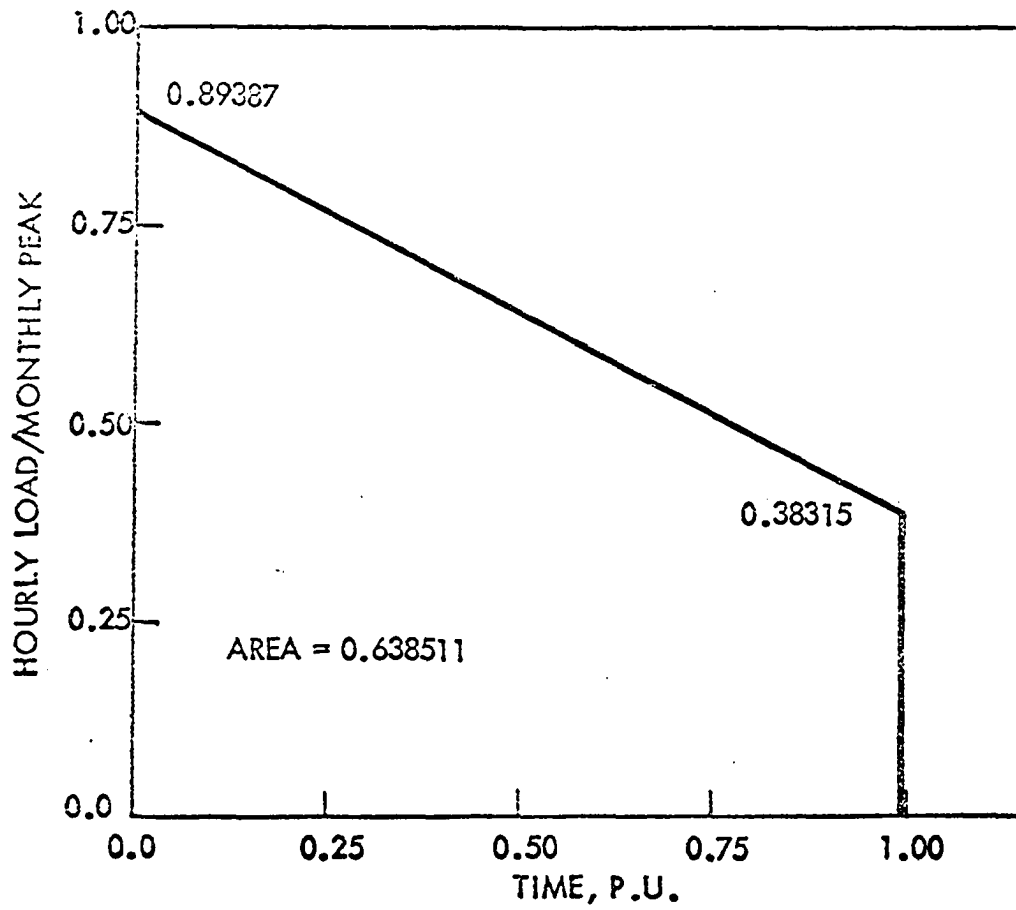


Fig. D.6. Monthly load duration curve for April

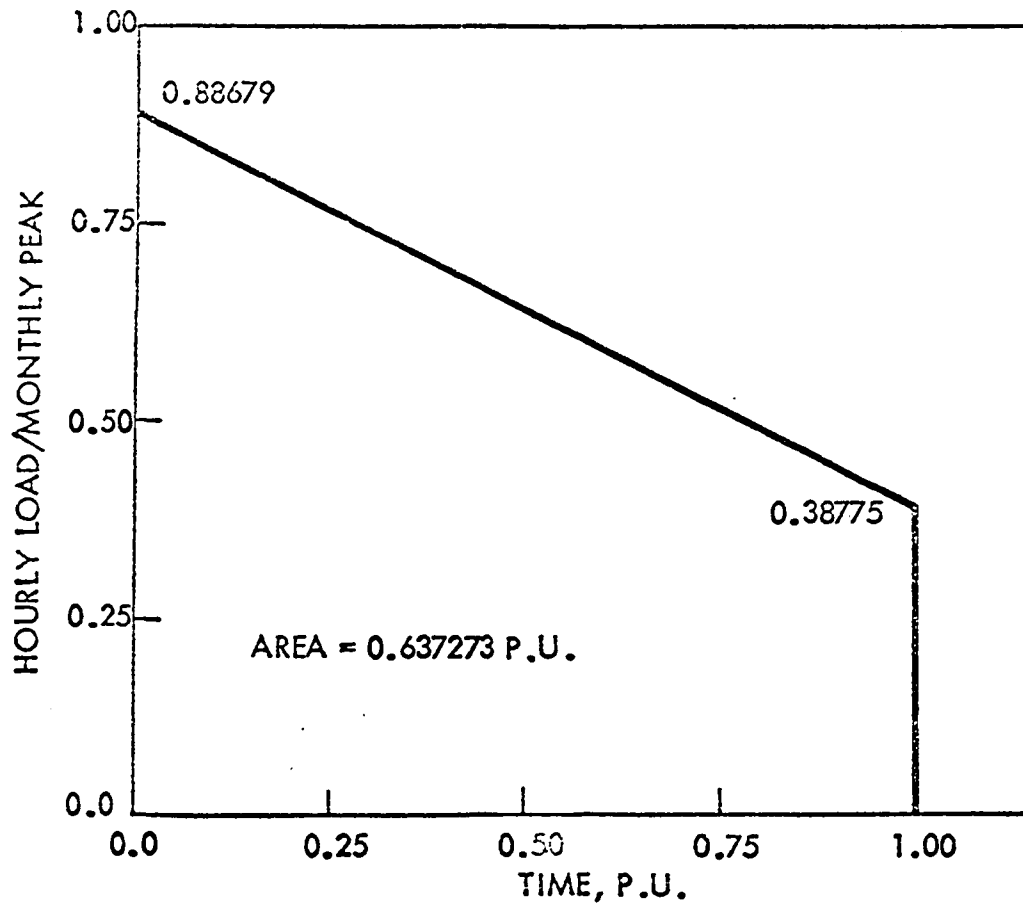


Fig. D.7. Monthly load duration curve for May



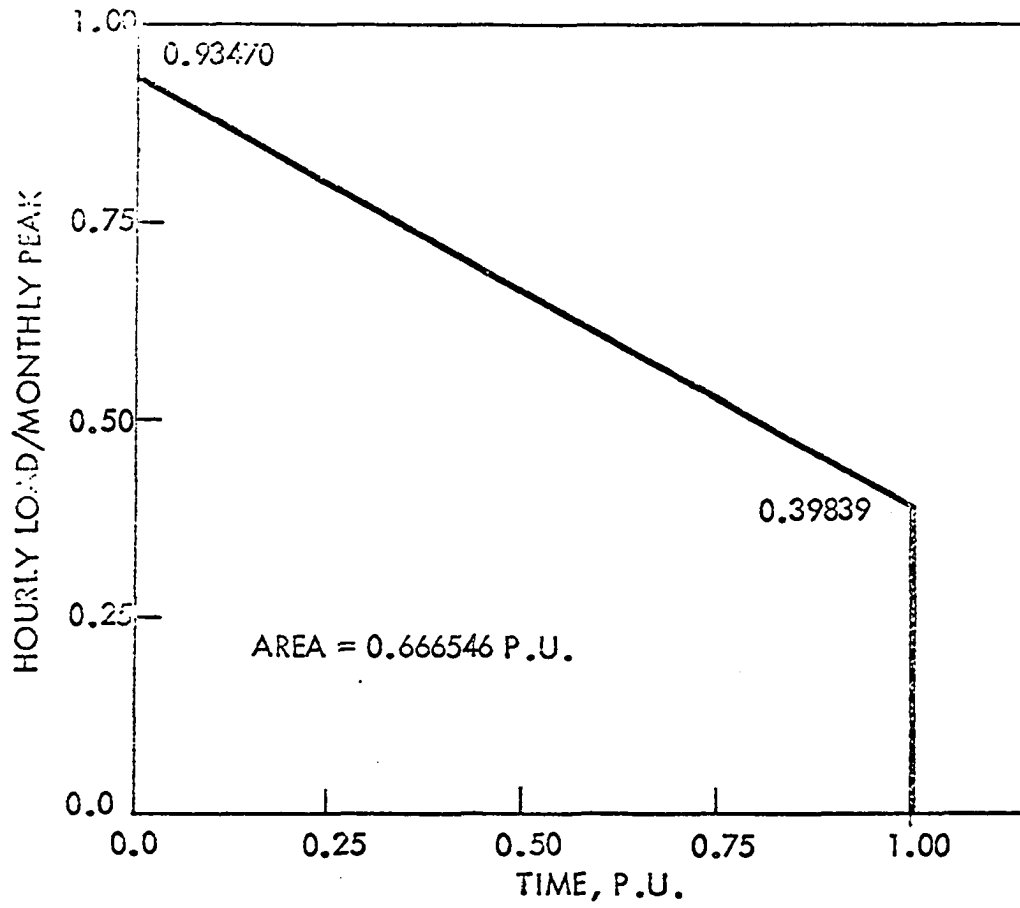


Fig. D.8. Monthly load duration curve for June

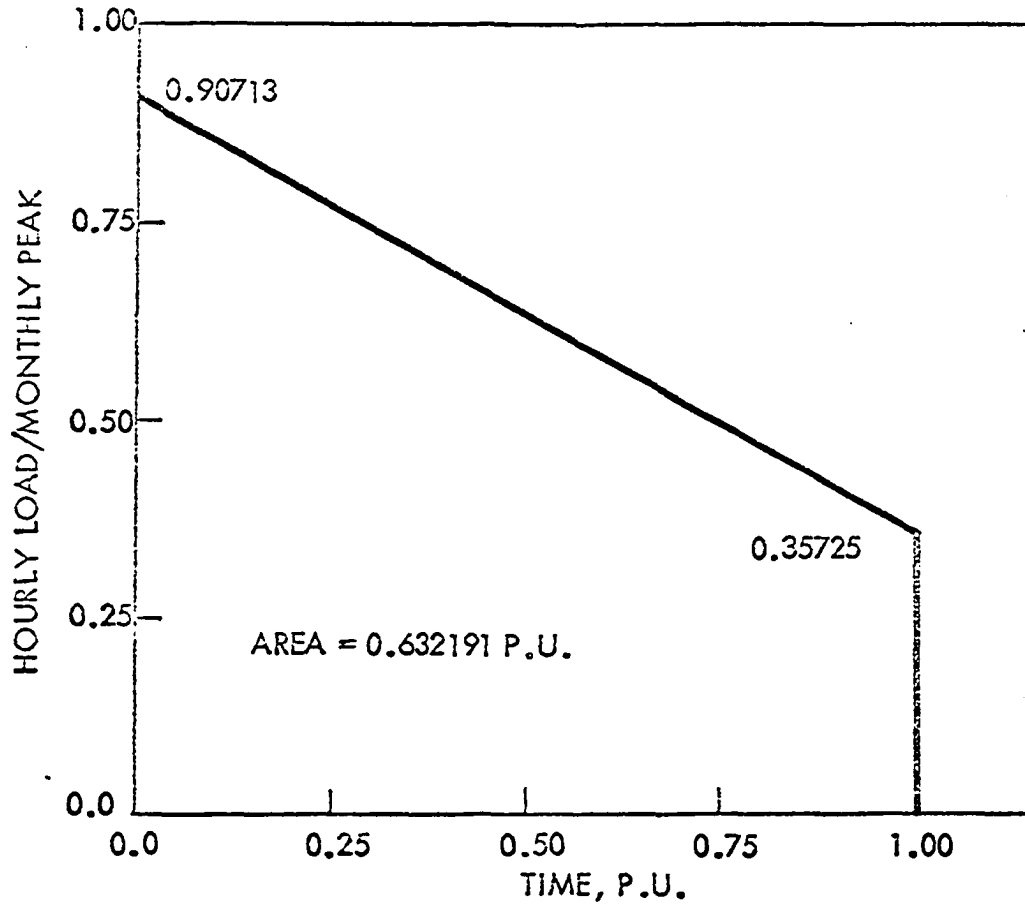


Fig. D.9. Monthly load duration curve for July

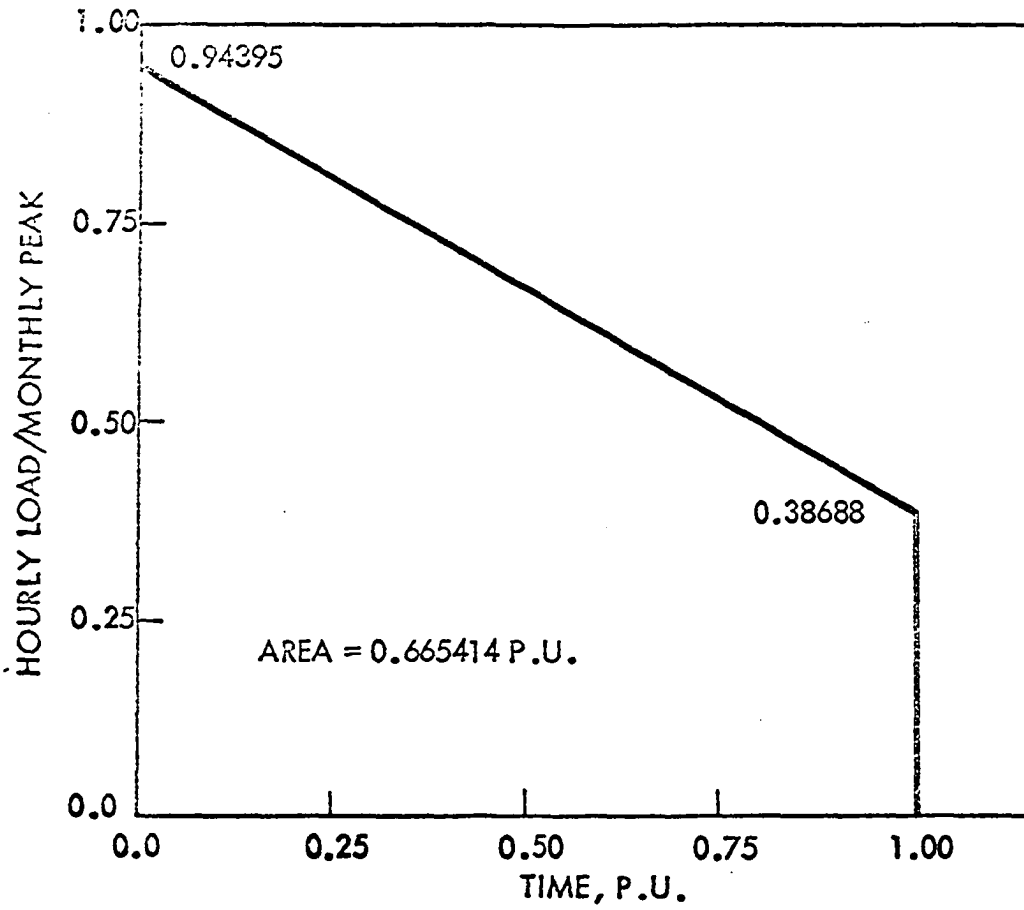


Fig. D.10. Monthly load duration for August

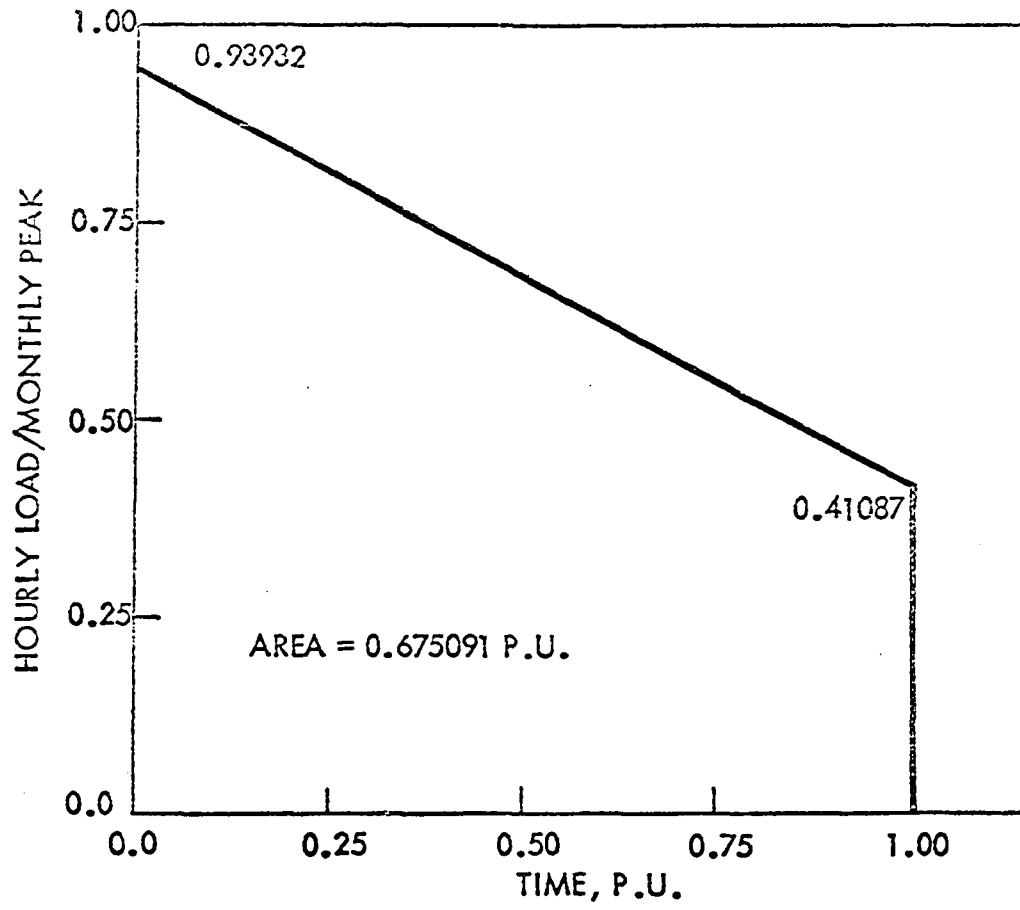


Fig. D.11. Monthly load duration curve for September

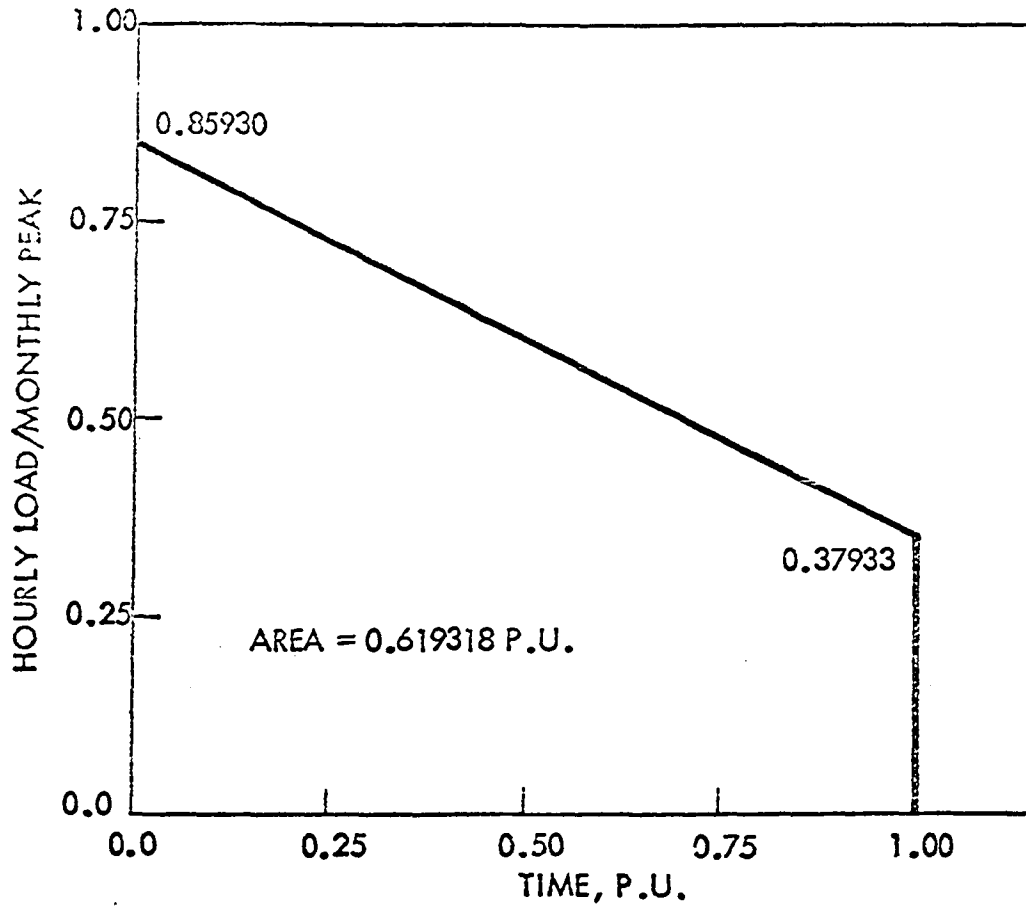


Fig. D.12. Monthly load duration curve for October

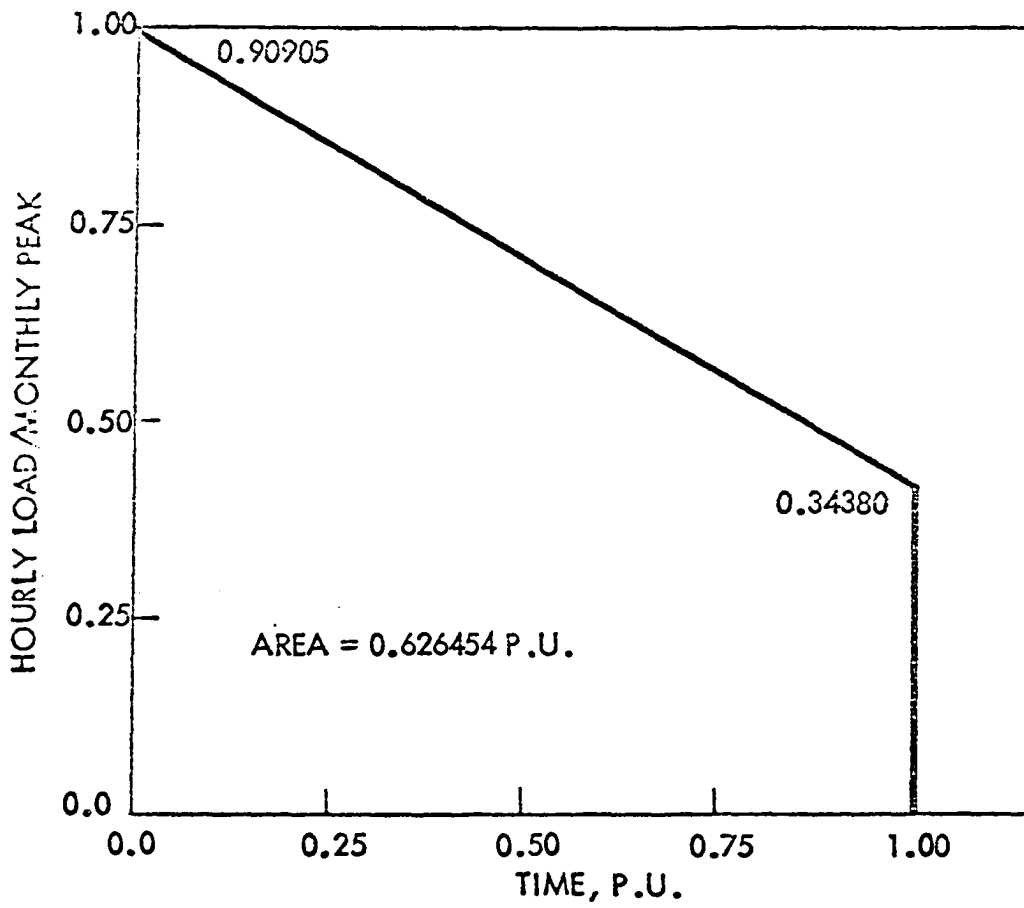


Fig. D.13. Monthly load duration curve for November

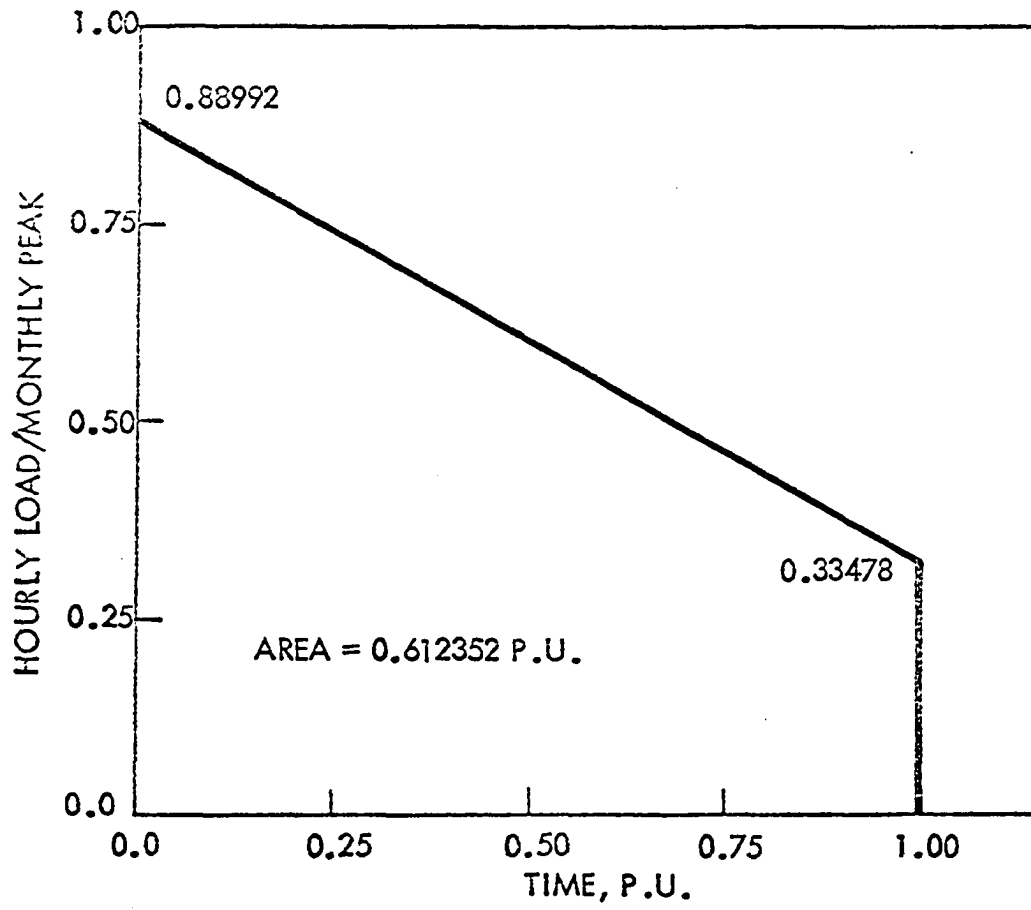


Fig. D.14. Monthly load duration curve for December

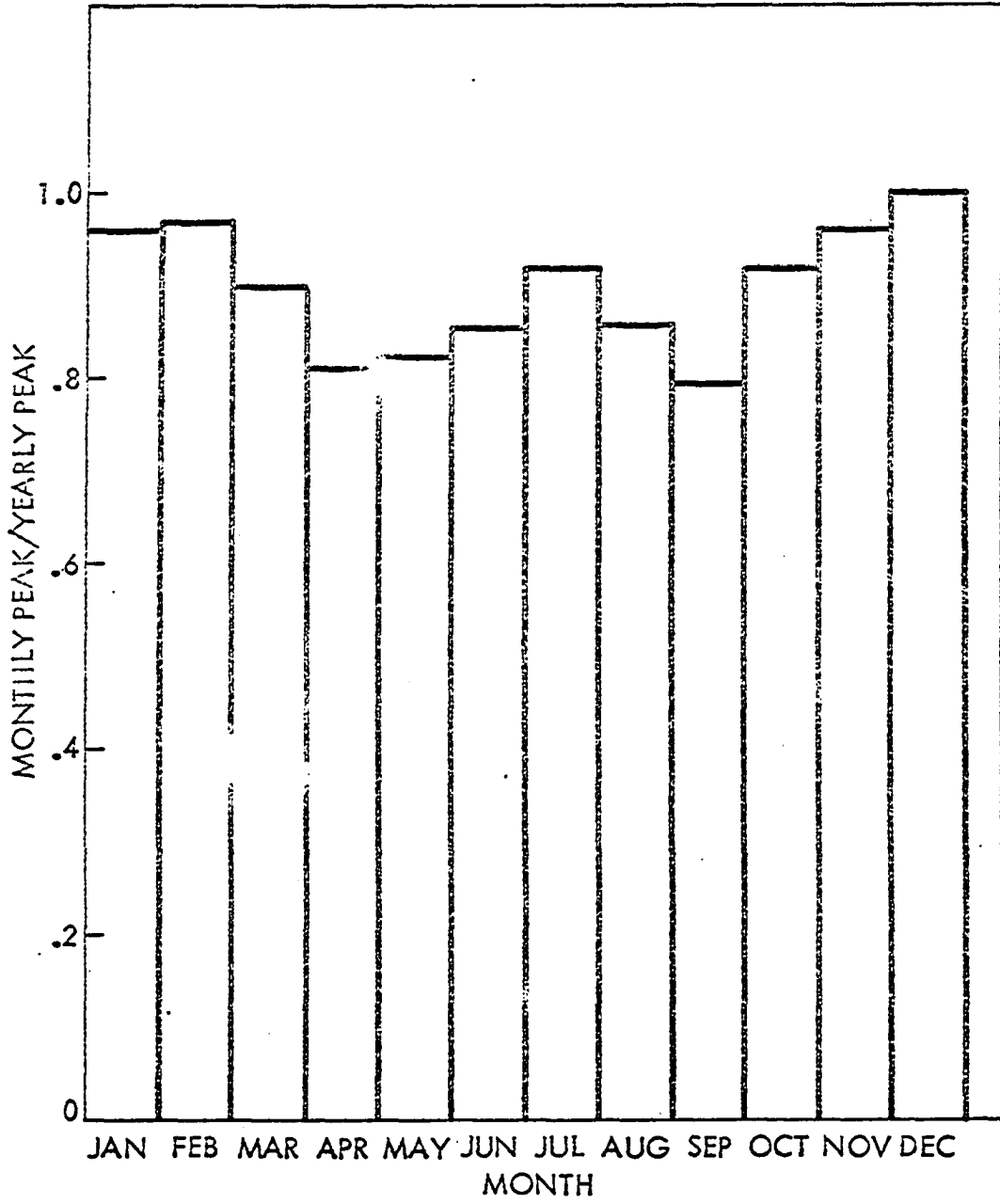


Fig. D.15. P.U. monthly peak for 1967 for the Pool data



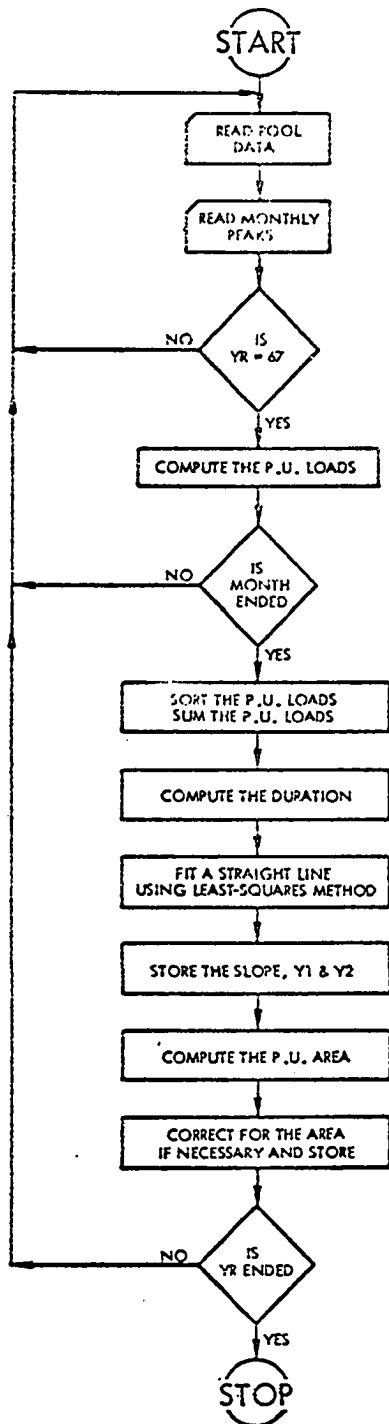


Fig. D.16. Subroutine Loader

```

*****
LOAD DURATION PROGRAM USING STRAIGHT LINE APPROXIMATION
*****
DIMENSION IC(6), IO(24), IP(744), ASUM(372), YY1(12), YY2(12)
DIMENSION AREA(12), BB(372), AA(372), INTVPK(12)
DIMENSION SL(12), AX(744), AINVRT(12), M(2)
YEAR=0
J=1
ISW=1
DO 650 I=1, 372
AA(I)=0.0
BB(I)=0.0
100 K=0
INEM=0
ID=1
GO TO (200, 300), ISW
200 READ(1, 10) (IC(I), I=1, 4)
10 FORMAT(3I2, 2X, I2, 10X)
IF(IC(3).NE.66) GO TO 200
IF(IC(1).NE.12) GO TO 200
IF(IC(2).NE.31) GO TO 200
IF(IC(4).NE.24) GO TO 200
ISW=2
300 IJ=24*(ID-1)
DO 400 L=1, 24
READ(1, 20, END=1001) (IC(I), I=1, 6)
20 FORMAT(5I2, 6X, I4)
IF(ID.NE.1) GO TO 240
M(1)=IC(1)
M(2)=IC(1)
240 M(2)=IC(1)
JK=IC(5)
IF(INEM.GT.IC(6)) GO TO 401
INEM=IC(6)
401 IP(IJ+L)=IC(6)
AX(IJ+L)=IJ+L
K0=IJ+L

```

Program 4. Load duration program

```

400 CONTINUE
   ID=ID+1
   IF(M(1).EQ.M(2)) GO TO 300
1001 CONTINUE
   INTVPK(J)=INEW
   CALL LFSORT(IP,KQ,AX)
   SUM=0.
   DO 500 I=1,KQ,2
   K=K+1
   ISUM=IP(I)+IP(I+1)
   ASUM(K)=FLOAT(ISUM)/(2.*FLOAT(INEW))
500 SUM=SUM+2.0*ASUM(K)
   AREA(J)=SUM/FLOAT(KQ)
   WRITE(3,515) AREA(J)
515 FORMAT(' ',40X,' AREA = ',F10.6)
   AK=K
   KS=1
   ASUM(K+1)=0.0
   DO 600 I=2,K
   IF(ASUM(I-1).EQ.ASUM(I)) GO TO 770
   BB(KS)=ASUM(I-1)
   AA(KS)=FLOAT(I-1)/AK
   GO TO 880
770 IF(ASUM(I).EQ.ASUM(I+1)) GO TO 600
   BB(KS)=ASUM(I)
   AA(KS)=FLOAT(I)/AK
880 KS=KS+1
600 CONTINUE
C   FITTING OF A STRAIGHT LINE USING LEAST SQUIRE METHOD
   SUM2=0.
   SUM3=0.
   SUM4=0.
   SUM5=0.
   DO 700 I=1,KS
   SUM2=SUM2+AA(I)
   SUM3=SUM3+BB(I)

```

Program 4 (Cont.)

```

SUM4=SUM4+AA(I)*AA(I)
700 SUM5=SUM5+AA(I)*BB(I)
XAV=SUM2/FLOAT(KS)
YAV=SUM3/FLOAT(KS)
A1 = SUM4-SUM2*SUM2/FLOAT(KS)
A2 = SUM5-SUM2*SUM3/FLOAT(KS)
B=-A2/A1
A=-(B*XAV)+YAV
IDY=ID-1
WRITE(3,808) IDY,XAV,YAV,A1,A2,B,A,M(1)
808 FORMAT(' ',20X,I10,6F10.5,I10)
C THE EQUATION OF LINE IS Y=A+BX
SL(J)=B
Y1=A
Y2=A+B
AREA1=(Y1+Y2)/2.0
DEL=AREA(J)-AREA1
IF(DEL) 860,900,860
860 Y1=Y1+DEL
Y2=Y2+DEL
900 YY1(J)=Y1
YY2(J)=Y2
WRITE(3,808) J,Y1,Y2,SL(J),AREA1,AREA(J),DEL,M(1)
WRITE(2,405) J,Y1,Y2,SL(J),AREA1
405 FORMAT(I10,4F12.6)
J=J+1
IF(IYEAR.GT.INEW) GO TO 105
IYEAR=INEW
105 IF(J.NE.13) GO TO 100
DO 450 I=1,12
AINVRT(I)=FLOAT(INTVPK(I))/FLOAT(IYEAR)
WRITE(3,505) INTVPK(I),AINVRT(I)
505 FORMAT(' ',30X,I10,20X,F10.5)
450 CONTINUE
WRITE(2,447) (AINVRT(I),I=1,12)
447 FORMAT(12F7.5)
Program 4 (Cont.)

```

1000 STOP  
END

159a

Program 4 (Cont.)

## XVI. APPENDIX E. SYSTEM RELIABILITY MEASUREMENTS

In this appendix, the three methods of computing system reliability will be discussed and a comparison between them will be made.

## The Loss of Load Probability (LOLP)

Several methods are suggested as a means of evaluating the LOLP. Three methods will be derived with a small example to illustrate how each works.

The first method

Consider the forced outage rate,  $p$ , for each unit of a system of  $n$  units of equal sizes. The probability of an outage not occurring,  $q$ , will be

$$q = 1 - p \quad (\text{E.1})$$

Using the binomial expansion (42) or the multiplication law for independent events, the probability of a certain sequence of  $r$  outages on our system will be  $p^r q^{n-r}$ .

Also using the rule of combinations and permutations, there are  $\frac{n!}{r!(n-r)!}$ , written as  $\binom{n}{r}$ , equally probable sequences in which  $r$  outages can occur on an  $n$ -unit system.

The probability of outages of various combinations of  $n$  units are given by the binomial expansion

$$(q + p)^n = q^n + \binom{n}{1} q^{n-1} p + \binom{n}{2} q^{n-2} p^2 + \dots + \binom{n}{r} q^{n-r} p^r + \dots + p^n \quad (\text{E.2})$$

where  $r$  is the number of machines out of service at the same time due to forced outages.

Let a group of 4 units, each of a capacity equal to 20 MW be connected to a second group of 3 units each of 30MW capacity. Assume  $P = .02$  and  $q = .98$ . Using equation 2, we will have, for the first group, the following outage probabilities.

Table E.1. Outage probability for the first group

No. of units out of service	Outage capacity MW	Probability of outage
0	0	0.92236800
1	20	0.07529500
2	40	0.00230400
3	60	0.00003130
4	80	0.00000016

A similar table will be computed for the second group as follows.

Table E.2. Outage probability for the second group

No. of units out of service	Outage capacity MW	Probability of outage
0	0	0.9411920
1	30	0.0576239
2	60	0.0011760
3	90	0.0000080

Now to find the probability of having  $n_1$  units of the first group and the same time having  $n_2$  units of the second group out of service, we use the multiplication rule which states (41):

$$P(E_1 \text{ and } E_2) = P(E_1)P(E_2) \quad (\text{E.3})$$

where both  $E_1$  and  $E_2$  are independent events. The the probability of having zero MW on outage of the combined system will be

$$\begin{aligned} P(O_1 \text{ and } O_2) &= P(O_1) \cdot P(O_2) \\ &= (.922368)(.941192) \\ &= 0.868125 \end{aligned} \quad (\text{E.4})$$

where  $O_1$  refers to zero MW for the first group and  $O_2$  refers to zero MW for the second group. Now we can say that the probability of having all units on service is equal to 0.868125.

Should we find in our table, two probabilities or more for a specific MW value, we have to use the second rule of probability theory which states

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) \quad (\text{E.5})$$



This is known as the addition law of probability. For example, the probability of losing 60 MW of service could be achieved by losing 3 units of the first group or 2 units of the second.

Then we can say

$$P(3_1 \text{ or } 2_2) = P(3_1, 0_2) + P(0_1, 2_2) \quad (\text{E.6})$$

or

$$\begin{aligned} P(60\text{MW}) &= P(3_1)P(0_2) + P(0_1)P(2_2) \\ &= (.0000313)(.941192) + (.922368)(.001176) \end{aligned}$$

$$P(60) = 0.001115$$

The joint probability of the two systems will be as shown in the following two-dimensional array (22).

Table E.3. Outage probability for the combined system

		Three 30MW units			
		0	1	2	3
Four 20 MW units	0	0.868125	0.053150	0.000003	.0000074
	1	0.070868	0.004339	0.000089	-----
	2	0.002168	0.000133	0.000003	-----
	3	0.000030	0.000002	-----	-----
	4	-----	-----	-----	-----

Finally, we may compute the following table of outage probabilities and cumulative probabilities for the 120 MW system.

Table E.4. Outage probability and cumulative probability for the combined system

Outage $\theta$ MW	Probability of outage = $\theta$ MW	Probability of outage $\geq \theta$ MW
0	0.868125	1.000000
10	-----	0.131875
20	0.070868	0.131875
30	0.053150	0.061007
40	0.002168	0.007857
50	0.004339	0.005689
60	0.001115	0.001350
70	0.001133	0.000235
80	0.000089	0.000102
90	0.000002	0.000013
100	0.000003	0.000011
110	-----	0.000008
120	-----	-----

If another group is added to the system, the procedure is repeated and a similar table will be obtained.

This method was programmed and tested for the Iowa Pool during 1969. Some comments on the method are:

1. The system should be grouped into subgroups of equal size with equal forced outage rates for this method of computation. This is not feasible in a real

system since it consists of different size units, all with different forced outage rates.

2. The method requires a large memory and relatively long computation time.
3. The method is not conveniently arranged to handle special units of different sizes.
4. The method requires sorting after every meshing of a new subgroup which further increases the computation time.

#### The second method

This is based on the theory that any unit can be in service with a probability  $q$  and can be out of service with a probability  $p$  due to forced outages only. That is to say, that the unit has a zero MW outage  $q$  per unit of the time and has an outage equal to its capacity  $p$  per unit of the time, keeping in mind that the sum of  $p$  and  $q$  is always unity. The following example will explain how the method works.

Consider the system of 3 units of 30 MW each as before with  $p = 0.02$  and  $q = 0.98$ . Adding the first unit, we compute the outage probability as follows:

<u>MW Outage</u>	<u>Probability</u>
0	0.980
30	0.020

To add the second unit, we consider that this unit is in service (zero MW outage) with a probability 0.98. Using the

multiplication rule, we will have:

<u>MW Outage</u>	<u>Probability</u>
0 + 0	$P(0)P(0) = 0.98 \times 0.98 = 0.960399$
30 + 0	$P(30)P(0) = 0.02 \times 0.98 = 0.01960$

Next, consider the second unit is on outage with a probability 0.02, that is the MW outage is equal to 30 MW.

<u>MW Outage</u>	<u>Probability</u>
0 + 30	$P(0)P(30) = 0.98 \times 0.02 = 0.01960$
30 + 30	$P(30)P(30) = 0.02 \times 0.02 = 0.00040$

The two results now can be joined as shown in Table E.5.

Table E.5. Outage probability for two 30MW units from the previous two steps

MW Outage	Probability
0	0.960399
30	0.019600
30	0.019600
60	0.000400

Using the addition rule of probability theory, we can say that

$$\begin{aligned}
 P(30 \text{ or } 30) &= P(30) + P(30) \\
 &= 0.0196 + 0.0196 = 0.0392
 \end{aligned}$$

and we will have the result shown in Table E.6.

Table E.6. Outage probability of two 30 MW units

MW Outage	Probability
0	0.960399
30	0.039200
60	0.000400

Now the third unit is added as before, and the result is shown below in two steps. The first step is to consider the unit in service.

<u>MW Outage</u>	<u>Probability</u>
0 + 0	$0.960399 \times 0.98 = 0.941192$
30 + 0	$0.03920 \times 0.98 = 0.038416$
60 + 0	$0.00040 \times 0.98 = 0.000392$

The second step is to consider the unit out of service.

<u>MW Outage</u>	<u>Probability</u>
0 + 30	$0.960399 \times 0.02 = 0.019208$
30 + 30	$0.0392 \times 0.02 = 0.000784$
60 + 30	$0.0004 \times 0.02 = 0.000008$

Now the combined system will be as shown in Table E.7.

Table E.7. Outage probability for the second group

MW Outage	Probability	Cumulative probability
0	= 0.941192	1.0000000
30	0.038416 + 0.019208 = 0.057624	0.0588086
60	0.000392 + 0.000784 = 0.001176	0.0011840
90	= 0.000008	0.0000080

which is the same result obtained by the first method as given in Table E.2.

In a system like the Iowa Pool with about 70 units, the units are added one at a time, each with its own capacity and its outage forced rate.

It is clear that this method is superior to the first one for the following reasons:

1. It is easy to comprehend and easy to program.
2. Each unit is added with its own capacity and its own forced outage rate, giving more accurate results than the previous method.
3. It is faster than the other method and has lower memory capacity requirement.

### The third method

This method is similar to the second method except for some modifications which considerably cut the time of computation.

As in the second method, the units will be added individually, but the main difference is we will work with cumulative

probability rather than with the probability. Since the cumulative probability is of greatest interest, this method will be more useful than Method 2.

The following example will illustrate the method. Consider the same example of 3 units each of 30MW capacity with  $p = 0.02$ . As before, adding the first unit we will have the following table of cumulative probability:

<u>MW Outage (<math>\theta</math>)</u>	<u>Probability of <math>\theta</math> or more (Cum. Prob.)</u>
0	1.0000
30	0.0200

Now add the second unit in two steps as before. First the unit is in service with  $q = 0.98$ .

<u>MW Outage (<math>\theta</math>)</u>	<u>Cumulative probability</u>
0 + 0	1.0 x .98 = 0.9800
30 + 0	.02 x .98 = 0.0196

Next the second unit is out of service with  $p = 0.02$ .

<u>MW Outage (<math>\theta</math>)</u>	<u>Cumulative probability</u>
0 + 30	1.0 x 0.02 = 0.0200
30 + 30	0.02 x 0.02 = 0.0004

Now to combine the two steps, we must explain what we mean by cumulative probability. Usually we say that it means the probability of having a variable equal or exceed a specific value. Thus, we say that the cumulative probability of having an outage of zero value or more will be the sum of the cumulative probabilities of having outage of zero or more from the first step and the cumulative probability of having outage of the

least MW outage of the second step, i.e., the cumulative probability (P) of having 0 or more will be

$$\begin{aligned} P(0 \text{ or more}) &= P(0)_1 + P(\text{the least value of} \\ &\quad \text{step 2} = 30) \\ &= 0.98 + 0.02 = 1.000 \end{aligned} \quad (\text{E.7})$$

$$\begin{aligned} P(30 \text{ or more}) &= P(30)_1 + P(30)_2 \\ &= 0.0196 + 0.02 = 0.0396 \end{aligned} \quad (\text{E.8})$$

$$\begin{aligned} P(60 \text{ or more}) &= P(60)_2 \\ &= 0.0004 \end{aligned} \quad (\text{E.9})$$

Then we will have the following table.

Table E.8. Cumulative probability for two 30 MW units

MW Outage	Cumulative probability
0	1.00000
30	0.03960
60	0.00040

Now add the third unit as before in two steps.

<u>MW Outage</u>	<u>Cumulative probability</u>
0 + 0	1.0 x 0.98 = 0.98000
30 + 0	0.0396 x 0.98 = 0.038808
60 + 0	0.0004 x 0.98 = 0.000392



<u>MW Outage</u>	<u>Cumulative probability</u>
0 + 30	1.0 x 0.02 = 0.020000
30 + 30	0.0396 x 0.02 = 0.000792
60 + 30	0.0004 x 0.02 = 0.000008

Since the cumulative probability of MW outage equaling zero or more is 0.98 and the cumulative probability of MW outage equal to 30 or more from the next table is equal to 0.02, then the cumulative probability of MW outage equal to zero or more will be the sum of both values, i.e.,

$$P(0 \text{ or more}) = 0.98 + 0.02 = 1.0000$$

$$P(30 \text{ or more}) = 0.0388086 + 0.02 = 0.0588086$$

$$P(60 \text{ or more}) = 0.000392 + 0.000792 = 0.001184$$

$$P(90 \text{ or more}) = 0.000008$$

The following table will summarize the above calculations

Table E.9. Cumulative probability for the second group

<u>MW Outage</u>	<u>Cumulative probability</u>
0	1.0000000
30	0.0588086
60	0.0011840
90	0.0000080

which is the same result obtained by the second method as shown in Table E.7.

In order to get the probability of having exactly an outage of specific value of MW, we have simply to subtract two consecutive cumulative probabilities of this MW and the second value in the table and replace the cumulative of this MW by the difference, i.e.,  $p(o)$

$$p(o) = P(o) - P(30)$$

$$= 1.0000 - 0.058808 = 0.941192$$

$$p(30) = 0.058808 - 0.001184 = 0.057624$$

$$p(60) = 0.001184 - 0.0000080 = 0.001176$$

$$p(90) = 0.0000080$$

which agrees with the previous results.

This method has all the advantages of the second method plus the vital consideration that no sorting is needed in any case. It also requires less memory capacity and is much faster than the others.

Each of the three methods were programmed and written in FORTRAN IV using IBM 360/65 computer. A list of results for the Iowa Pool system are given in Table E.16 at the end of this appendix. The third method was used for these calculations although each program gives the same results.

### Evaluating the risk level

So far, we have discussed some methods to compute either the probability or the cumulative probability of MW outages. We have not mentioned how to measure the index of reliability or the risk level. In the following pages, we will derive some mathematical formulas to measure this index.

All the methods developed to measure the reliability index are based on evaluating the probability of having negative margin, i.e., generation deficiency. Several methods are proposed on how to compute this probability.

Computation of risk index using a series of daily peak loads

The cumulative probability table is used in calculating the LOLP given a series of daily peak loads for the period under study (53). To explain the method, let the 120 MW system given in Table 4 be considered. Assume that the year has 26 intervals each of 10 days' duration. Let the peak loads in the first three days of a particular interval be 55, 65, 90 MW respectively. Let the interval peak be 100 MW. The reserve will be given by

$$\begin{aligned} \text{Reserve} &= \text{installed capacity} - \text{interval peak} && \text{(E.10)} \\ &= 120 - 100 = 20 \text{ MW} \\ \% \text{ Reserve} &= \frac{20}{120} \times 100 = 16.66\% \end{aligned}$$

To find the LOLP for the first day, we have to know what value of outage will make the available capacity to be equal to or less than the load, i.e.,  $65 (= 120 - 55)$  MW. An outage of greater than  $120 - 55 = 65$  MW will result in a loss-of-load.

From Table E.4, an outage of 70 MW must be counted as it is the entry equal to or greater than 65. Thus, the LOLP will be 0.000235 days/day.

For the second day, the LOLP will be the cumulative probability of an outage of  $120 - 65 = 55$  MW. From Table E.4, an

outage of 60 MW will result in a loss-of-load, and the LOLP will be 0.005687 days/day. Similarly, for the third day, the LOLP will be 0.061007 days/day. If all the ten days in this interval are treated the same way, the sum of the LOLP of every day will be the LOLP of that interval expressed in days/10 days. The sum of the LOLP for the 26 intervals will give the annual LOLP expressed in days/year. This risk index is compared with the threshold risk level chosen by the planner. This risk level is of great importance and care should be taken in selecting this value. For a system with important loads, this value is often taken as 0.1 or 1 day per 10 years. For other systems, it could be set equal to 0.2 or 1 day per 5 years. This choice will affect the planning of generation additions as well as spinning reserve, the scheduled outages and the cost of the whole plan.

This method is very simple but it does not give the spinning reserve. It takes a relatively long time to compute and does not evaluate load forecast deviations.

#### Computation of risk index using the probability distribution function

This method is based on considering the monthly loads  $Y$  is normally distributed with a mean  $\bar{Y}$  and a standard deviation  $\sigma_Y$  (54). This is shown in Fig. E.1 for a typical month where  $Y$  is the load and  $p(Y)$  is its probability density. Now the peak will be

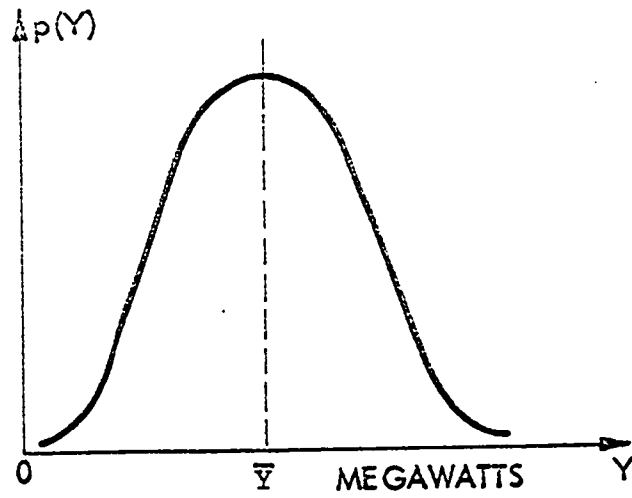


Fig. E.1. Probability of monthly average load

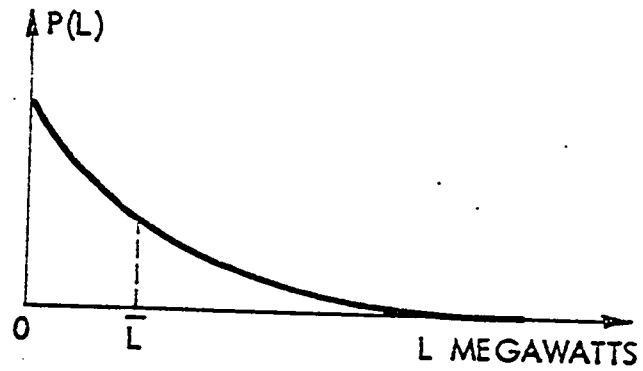


Fig. E.2. Probability density for loss of capacity

$$Y_m = \bar{Y} + 1.89\sigma_Y \quad (\text{E.11})$$

where 1.89 is the first-order statistic for a 21-day month (excluding weekends).

We know that the assumption of normality and the use of full distribution is supported by the central limit theorem of probability theory which states that when a collection of non-normal events are combined, their sum tends to be normal. That is the case for power systems. For example, the Pool Data is the sum of six random loads of the six companies. Thus, the cumulative probability of loss of load may be considered to be a continuous distribution with a mean  $\bar{L}$  and a standard deviation  $\sigma_L$  where

$$\bar{L} = \sum p_i C_i, \quad i = 1, 2, \dots, n \quad (\text{E.12})$$

$$\sigma_L^2 = \sum p_i (1-p_i) C_i^2, \quad i = 1, n \quad (\text{E.13})$$

where  $p_i$  is the forced outage rate for unit  $i$  and  $C_i$  is the capacity of that unit,  $L$  is the load loss in megawatts.

Fig. E.2 shows the probability density function of loss of capacity which is the derivative of the cumulative probability distribution function. This exponential shape of curve is typical of that used in reliability studies.

Now the available capacity of  $X$  megawatts is given by

$$X = I - L \quad (\text{E.14})$$

megawatts where  $I$  is the installed capacity. The probability density function of the available capacity will be as shown in Fig. E.3 with a mean

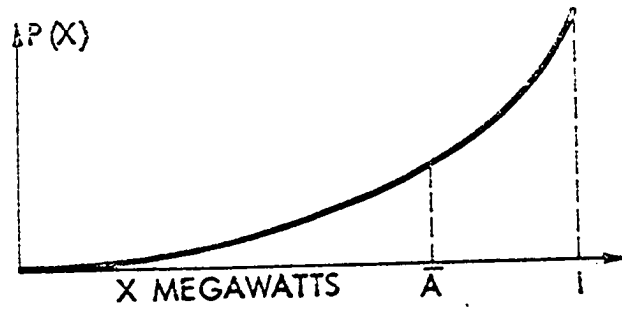


Fig. E.3. Probability density for the available capacity

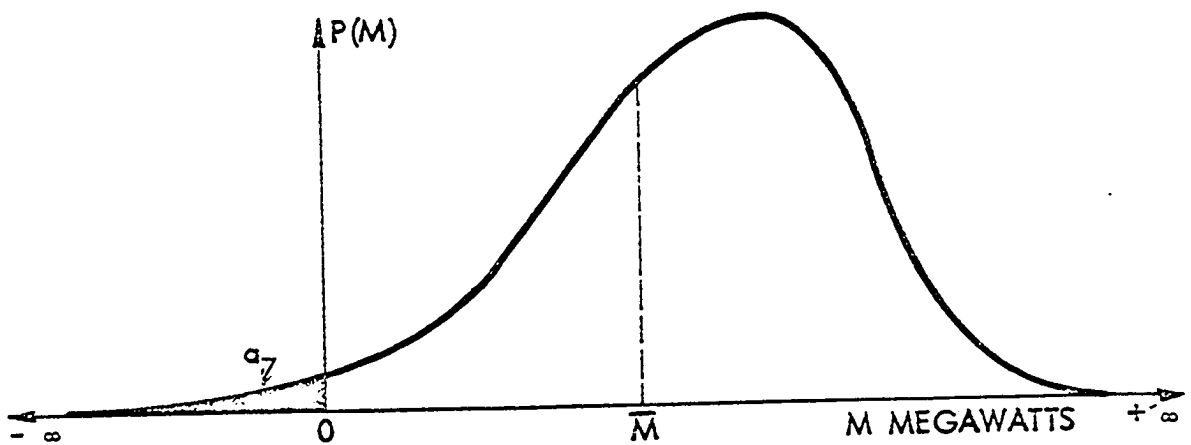


Fig. E.4. Margin of available capacity distribution

$$\bar{A} = I - \bar{L} \quad (\text{E.15})$$

and a standard deviation  $\sigma_L$ .

The margin M will be given by

$$M = X - Y \quad (\text{E.16})$$

For this method, we assume that both the load and the available capacity are normally distributed, then the mean  $\bar{M}$  will be (55)

$$\bar{M} = \bar{A} - \bar{Y} \quad (\text{E.17})$$

and

$$\sigma_M^2 = \sigma_X^2 + \sigma_Y^2 \quad (\text{E.18})$$

But we notice that the available capacity distribution is not normal and equations E.17 and E.18 do not apply. A more approximate statistical mathematics should be developed to solve this problem.

The probability density function of the margin M will be as shown in Fig. E.4. The probability of having negative margin (generation deficiency) will be the area from  $-\infty$  to 0 (the area "a" in Fig. E.4).

$$P(M < 0) = \int_{-\infty}^0 \frac{1}{\sqrt{2\pi}\sigma_M} e^{-(M-\bar{M})^2 / 2\sigma_M^2} dM \quad (\text{E.19})$$

Since the lower limit is  $-\infty$  the integral is an improper one and numerical integration would be terminated as the area contributions become sufficiently small for negative M. However, if we truncate the lower limit at  $M = \bar{M} - 4\sigma_M$ , a simpler procedure can be used. Using this approximation, if  $\bar{M} \geq 4\sigma_M$  there is no need to compute the risk index for that particular month.



This procedure gives the monthly risks. The annual risk will be the sum of the 12 monthly risks computed as before.

It should be mentioned that this method is not accurate due to the assumption of normality for both distributions. Also, numerical integration is a time-consuming process. However, a new method of calculating the exact marginal distribution will be given in the next method.

#### Computation of risk index using the multidimensional joint probability density

This method is based on using the continuous distributions of both loads and the available capacity as before. The only difference is that we approximate the probability of the available capacity by an exponential distribution. If we consider the mean of daily peak loads is normally distributed with a mean  $\bar{Y}$  and a standard deviation  $\sigma_Y$  for every month, the probability density function (p.d.f.) will be (42)

$$p(Y) = \frac{1}{\sqrt{2\pi}\sigma_Y} e^{-(Y-\bar{Y})^2/2\sigma_Y^2} \quad (\text{E.20})$$

i.e., Load  $Y$  is  $N(Y, \sigma_Y)$  as shown in Fig. E.1.

Now if we also consider the cumulative probability of the available capacity to have an exponential distribution, we can say that the probability distribution function will be

$$P(X) = ce^{mX}, \quad 0 < X < I \quad (\text{E.21})$$

This could be easily done by fitting a straight line to the logarithm of the cumulative probability against the available capacity using the least square technique (4). The straight line

equation will be

$$W = m(X - \bar{A}) + \bar{W} \quad (\text{E.22})$$

as shown in Fig. E.5, where  $W$  is the logarithm of the cumulative probability of  $X$ , the available capacity.

$\bar{A}$  and  $\bar{W}$  are the averages of the two values mentioned and  $m$  will be the slope of this straight line. Equation E.22 could be rewritten as

$$\begin{aligned} W &= mX + (\bar{W} - m\bar{A}) \\ &= mX + b \end{aligned} \quad (\text{E.23})$$

where  $b = (\bar{W} - m\bar{A})$ .

This straight line is shown in Fig. E.5. Now we can say that

$$P(X) = e^{mX+b} = ce^{mX} \quad (\text{E.24})$$

which is the same as equation E.21 with  $c = e^b$ . To get the probability density function of the available capacity, we differentiate equation E.2

$$p(X) = \frac{dP(X)}{dX} = cme^{mX} = ke^{mX} \quad (\text{E.25})$$

where  $k = cm$ .

Note  $p$  means p.d.f. while  $P$  means the cumulative probability.

On the assumption that both  $X$  and  $Y$  are mutually independent which is fortunately the case, we can write the joint probability function (j.p.f.) as the product of the two independent probability (55), i.e.,

$$p_{X,Y}(X,Y) = p_X(X)p_Y(Y) \quad (\text{E.26})$$

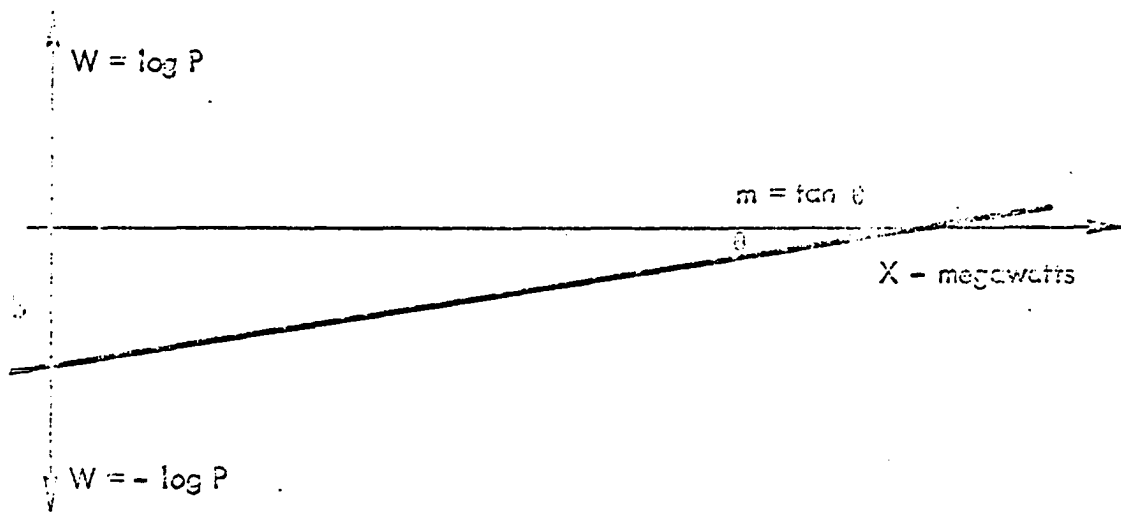


Fig. E.5. Straight line approximation to the cumulative probability of the available capacity

or

$$p_{X,Y}(X,Y) = \frac{K}{\sqrt{2\pi}\sigma_Y} e^{mX} e^{-(Y-\bar{Y})^2/2\sigma_Y^2} \quad (\text{E.27})$$

Consider the transformation

$$M = G(X,Y) \quad (\text{E.28})$$

$$N = H(X,Y), \quad (\text{E.29})$$

maps a region R of points in the XY plane into a region S of points in the MN plane in which both G and H are continuously differentiable. The Jacobian of this transformation will be

$$\frac{\partial(M,N)}{\partial(X,Y)} = \begin{vmatrix} \frac{\partial M}{\partial X} & \frac{\partial M}{\partial Y} \\ \frac{\partial N}{\partial X} & \frac{\partial N}{\partial Y} \end{vmatrix} \quad (\text{E.30})$$

If this Jacobian is not zero over R, there is a unique inverse transformation

$$X = g(M,N) \quad (\text{E.31})$$

$$Y = h(M,N) \quad (\text{E.32})$$

which takes each point (M,N) of S into a unique point (X,Y) in R, such that

$$G(g(M,N), h(M,N)) = M. \quad (\text{E.33})$$

$$H(g(M,N), h(M,N)) = N. \quad (\text{E.34})$$

The Jacobian of the inverse transformation is

$$\frac{\partial(X,Y)}{\partial(M,N)} = \left( \frac{\partial(M,N)}{\partial(X,Y)} \right)^{-1} \quad (\text{E.35})$$

and the change of variables in the double integral which evaluates the bivariate distribution function is done as follows:

$$\iint_R p_{X,Y}(X,Y) dXdY = \iint_S p_{X,Y}(g(M,N), h(M,N))$$

$$\left| \frac{\partial (X, Y)}{\partial (M, N)} \right| dM dN. \quad (E.36)$$

where S is the image of the region R under transformation. If S is a set in the MN plane and R is the set of all points in the XY plane that have images in S under the transformation, then the cumulative probability of M,N in S is equal to the cumulative probability of X,Y in R. Mathematically stated

$$P\{(M, N) \text{ in } S\} = P\{(X, Y) \text{ in } R\}. \quad (E.37)$$

or

$$\begin{aligned} P\{(M, N) \text{ in } S\} &= \iint_R p_{X, Y}(X, Y) dX dY. \\ &= \iint_S p_{X, Y}(g(M, N), h(M, N)) \\ &\quad \left| \frac{\partial (X, Y)}{\partial (M, N)} \right| dM dN \end{aligned} \quad (E.38)$$

But  $P\{(M, N) \text{ in } S\}$  can be written as

$$P\{(M, N) \text{ in } S\} = \iint_S p_{m, n}(M, N) dM dN, \quad (E.39)$$

and it follows that

$$p_{m, n}(M, N) = p_{X, Y}(g(M, N), h(M, N)) \left| \frac{\partial (X, Y)}{\partial (M, N)} \right|. \quad (E.40)$$

Since  $p_{m, n}(M, N) = p_m(M|N)p_n(N)$ , where  $p_m(M|N)$  is the conditional probability or

$$p(M) = \int p_m(M|N) p_n(N) dN \quad (E.41)$$

$$= \int p_{m, n}(M, N) dN, \quad (E.42)$$

and

$$g(N) = \int p_{m, n}(M, N) dM. \quad (E.43)$$

Now to get the probability of having a negative margin, we rewrite equations E.28 and E.29 as follows:

$$M = X - Y \quad (\text{E.44})$$

$$N = X \quad (\text{E.45})$$

The Jacobian will be equal to unity. Equations E.31 and E.32 will be

$$X = N \quad (\text{E.46})$$

$$Y = N - M \quad (\text{E.47})$$

$$p(M, N) = p_{X, Y}(N, N-M) \quad (\text{E.48})$$

and

$$p(M) = \int p_{X, Y}(N, N-M) dN = \int p_{X, Y}(X, X-M) dX \quad (\text{E.49})$$

On the other hand, if we set  $N = Y$  instead of equation E.45, we will have the Jacobian which will be still equal to unity and

$$\begin{aligned} p(M) &= \int p_{X, Y}(M+N, N) dN \\ &= \int p_{X, Y}(M+Y, Y) dY \end{aligned} \quad (\text{E.50})$$

Equations E.49 and E.50 can now be rewritten as

$$p(M) = \int p_X(X) \cdot p_Y(X-M) dX, \quad (\text{E.51})$$

and

$$p(M) = \int p_X(M+Y) p_Y(Y) dY \quad (\text{E.52})$$

Then

$$p(M) = \frac{k}{\sqrt{2\pi}\sigma_Y} \int_{a_1}^{b_1} e^{mX} \cdot e^{-(X-M-\bar{Y})^2/2\sigma_Y^2} dX \quad (\text{E.53})$$

or

$$p(M) = \int p_X(Y+M) \cdot p_Y(Y) dY \quad (\text{E.54})$$

i.e.,

$$p(M) = \frac{k}{\sqrt{2\pi}\sigma_Y} \int_{a_2}^{b_2} e^{m(Y+M)} e^{-(Y-\bar{Y})^2/2\sigma_Y^2} dY \quad (E.55)$$

Now we have obtained an expression for the probability density function for the margin of available capacity. In order to get the limits of both the integrations in equations E.53 and E.55 respectively, we must recognize that the integrand in these equations should not have a zero or negative value.

For the first integral, we know that  $X$  lies between 0 and  $I$  where  $I$  is the installed capacity, so  $a_1 = 0$ ,  $b_1 = I$ .

For the second integral, consider the following two inequalities:

$$\bar{Y} - 4\sigma < Y < \bar{Y} + 4\sigma \quad (E.56)$$

$$0 < X < I \quad (E.57)$$

Then  $a_2 = 0$ ,  $b_2 = I$  as before

If we consider equation E.55, we can rewrite it as

$$p(M) = \frac{ke^{mM}}{\sqrt{2\pi}\sigma_Y} \int_0^I e^{mY} e^{-(Y-\bar{Y})^2/2\sigma_Y^2} dY \quad (E.58)$$

$$= \frac{k}{\sqrt{2\pi}\sigma_Y} e^{mM} J \quad (E.59)$$

where  $J$  is the value of the integral.

In order to evaluate the probability of having a negative margin, we should evaluate  $J$ . Numerical methods may be used. In the following paragraph, a new method will be explained to evaluate this integral using transformation of variable which will result in a considerable saving in computation time (56).

Evaluation of integral (J).

To evaluate

$$J = \int_0^I e^{mY} \frac{e^{-(Y-\bar{Y})^2/2\sigma_Y^2}}{\sigma_Y} dY, \quad (\text{E.60})$$

we first let

$$Z = (Y-\bar{Y})/\sigma_Y \quad (\text{E.61})$$

such that

$$Y = Z\sigma_Y + \bar{Y} \quad (\text{E.62})$$

and

$$dY = \sigma_Y dZ \quad (\text{E.63})$$

Now replace Y by equation (E.62) with the limits of the integral J now taking the value

$$Y = I, Z = (I-\bar{Y})/\sigma_Y$$

and

$$Y = 0, Z = -\bar{Y}/\sigma_Y.$$

Then the integral in equation E.60 can be rewritten as

$$J = \int_{-\bar{Y}/\sigma_Y}^{(I-\bar{Y})/\sigma_Y} \frac{1}{\sigma_Y} e^{m(Z\sigma_Y + \bar{Y})} \frac{e^{-Z^2/2}}{\sigma_Y} dZ \quad (\text{E.64})$$

or

$$J = e^{m\bar{Y}} \int_{-\bar{Y}/\sigma_Y}^{(I-\bar{Y})/\sigma_Y} e^{m\sigma_Y Z} \frac{e^{-Z^2/2}}{\sigma_Y} dZ \quad (\text{E.65})$$

Now to evaluate such an integral using the usual numerical methods such as trapezoidal rule, Simpson's rule or Romberg's



method (57), (58) we have to choose a small interval and compute the infinitesimal area and add all these areas to obtain the total area under the curve.

Since  $e^{-Z^2/2}$  is symmetric about  $Z = 0$ , we can store half the ordinates corresponding to values of  $Z$  incremented by  $\Delta Z$  and these ordinates may be used in the successive computations. The only thing remaining is to find the ordinates of  $e^{-m\sigma_Y Z}$ , multiply it by the corresponding stored ordinates of  $e^{-Z^2/2}$  and then compute the small areas. Then we can say that  $J$  will have the value

$$J = \sigma_Y e^{m\bar{Y}} \cdot a \quad (\text{E.66})$$

where  $a$  is the integral value. Returning to equation E.55, we can write the p.d.f. of the margin as

$$p(M) = \frac{ka}{\sqrt{2\pi}\sigma_Y} e^{m\bar{Y}} \cdot e^{mM} \quad (\text{E.67})$$

The probability of having negative margin  $P(M < 0)$  will be

$$P(M < 0) = \frac{kae^{m\bar{Y}}}{\sqrt{2\pi}} \int_{M_1}^0 e^{mM} dM \quad (\text{E.68})$$

where the lower limit of the integral  $M_1$  must be evaluated. We know that the minimum margin or the maximum negative margin is when the load is maximum and the available capacity is minimum. Then  $M_1$  will be

$$M_1 = 0 - (\bar{Y} + 4\sigma_Y) = -(\bar{Y} + 4\sigma_Y) \quad (\text{E.69})$$

Under the transformation E.61,  $M_1$  will be

$$M_1 = -\left(4 + \frac{\bar{Y}}{\sigma_Y}\right) \quad (\text{E.70})$$

and

$$P(M < 0) = \frac{kae^{m\bar{Y}}}{\sqrt{2\pi}} \int_{-\left(4 + \frac{\bar{Y}}{\sigma_Y}\right)}^0 e^{mM} dM \quad (\text{E.71})$$

$$P(M < 0) = \frac{a}{\sqrt{2\pi}} Ce^{m\bar{Y}} \left\{1 - e^{-m\left(4 + \frac{\bar{Y}}{\sigma_Y}\right)}\right\} \quad (\text{E.72})$$

But  $Ce^{m\bar{Y}}$  is the cumulative probability of having the available capacity equal to  $\bar{Y}$ , the mean of daily peaks for that month is known and may be read directly from the cumulative probability table stored before. Then equation E.72 becomes

$$P(M < 0) = \frac{a}{\sqrt{2\pi}} P(\bar{Y}) \left\{1 - e^{-m\left(4 + \frac{\bar{Y}}{\sigma_Y}\right)}\right\} \quad (\text{E.73})$$

And in order to evaluate the risk value for any month, we should have:

1. The mean of daily peaks for that month ( $\bar{Y}$ ).
2. The standard deviation for that month ( $\sigma_Y$ ).
3. The value of the integral (J) given in equations E.64 and E.65.
4. The slope  $m$  of the exponential distribution of the available capacity.

A comparison of the three methods in computing the monthly risk value or the cumulative probability of having negative margin is in order.

1. The third method is more accurate than the first one due to the fact that in the first method, both distribution of load and available capacity are assumed to

be normal which is not true. But in the second method, both distributions were treated accurately, which will result in a marked improvement in evaluation of the risk value.

2. The last two methods require numerical integration.
3. The second method requires some corrections to compensate for the assumption that the available capacity distribution is normal which adds more computation time.
4. The first method requires the series of daily peak loads to be known for all the period under study which takes a relatively long time to compute the risk index.

Computation of the risk index using the cumulative probability of the available capacity

In this method, which is easy to handle, we assume that the mean of daily peaks in any month is normally distributed with a mean  $\bar{Y}$  and a standard deviation  $\sigma_Y$  as before. Also, we consider the cumulative probability of the available capacity to be exponentially distributed.

Stated mathematically,

$$P(Y) = \frac{1}{\sqrt{2\pi}\sigma_Y} e^{-\frac{(Y-\bar{Y})^2}{2\sigma_Y^2}}, \quad \bar{Y}-2\sigma_Y < Y < \bar{Y}+2\sigma_Y \quad (E.20)$$

$$P(X) = ce^{mX} \quad 0 < X < I \quad (E.21)$$

These two distributions are shown in Fig. E.1 and Fig. E.3

respectively.

The mean peak load  $Y$  varies between  $\bar{Y} + 2\sigma_Y$  as the upper bound and  $\bar{Y} - 2\sigma_Y$  as the lower bound. Assume these bounds be  $b$  and  $a$  respectively. The mean of the exponential distribution segment bounded by  $a$  and  $b$  as shown in Fig. E.6 is given as

$$\bar{X} = \frac{\text{The first moment of the distribution segment } (M_1)}{\text{The area of that distribution segment } (A)} \quad (\text{E.74})$$

where  $\bar{X}$  is the mean of all the loads in that segment. Since the first moment ( $M_1$ ) of any distribution is the expected values of the first power of the random variable which has that given distribution (56), we can write  $M_1$  as

$$M_1 = \int_a^b XP(X) dX \quad (\text{E.75})$$

Equation E.75 can now be written as

$$M_1 = c \int_a^b X e^{mX} dX \quad (\text{E.76})$$

or

$$M_1 = \frac{c}{m} (e^{mb} (b - \frac{1}{m}) - e^{ma} (a - \frac{1}{m})) \quad (\text{E.77})$$

Let  $k_1 = (b - \frac{1}{m})$  and  $k_2 = (a - \frac{1}{m})$ , then equation E.77 can be re-written as

$$M_1 = \frac{k_1}{m} \cdot P(X=b) - \frac{k_2}{m} \cdot P(X=a) \quad (\text{E.78})$$

The area under the segment of available capacity ( $A$ ) will be equal to

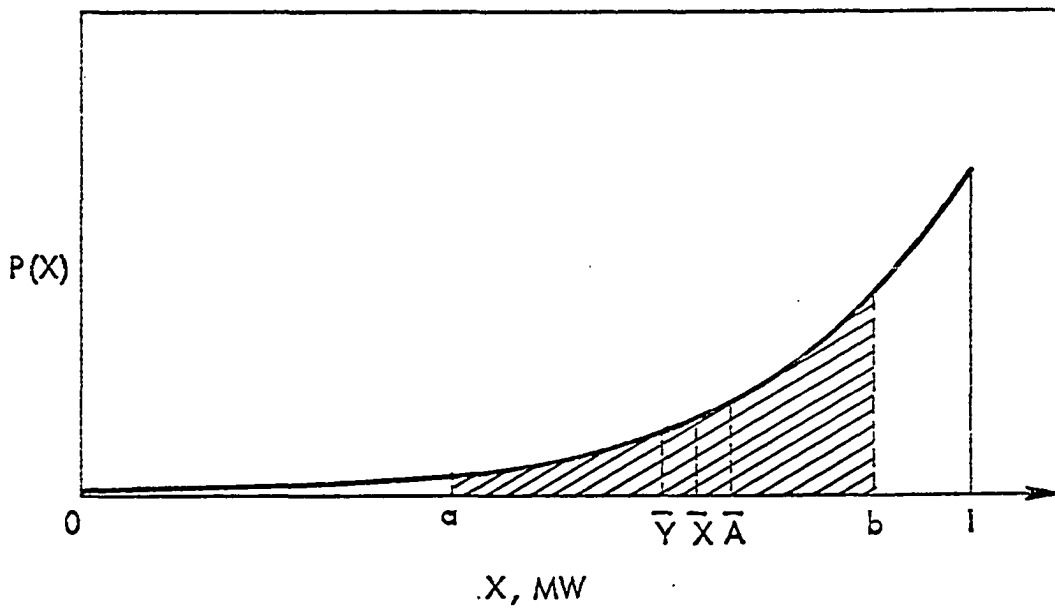


Fig. E.6. Probability density for the available capacity

$$\begin{aligned}
 A &= c \int_a^b e^{mX} dX & (E.79) \\
 &= \frac{c}{m} (e^{mb} - e^{ma})
 \end{aligned}$$

or

$$A = \frac{1}{m} (P(X=b) - P(X=a)) \quad (E.80)$$

Now the mean of the distribution segment will be

$$\bar{X} = \frac{k_1 \cdot P(X=b) - k_2 \cdot P(X=a)}{P(X=b) - P(X=a)} \quad (E.81)$$

The average availability for that month ( $\bar{V}$ ) or the risk index will be given as

$$\bar{V} = P(\bar{X}) \quad (E.82)$$

and equation E.82 gives a measure of the risk index for that month.

The following example will illustrate this method of measuring the risk index.

For the Iowa Pool, the cumulative probability distribution was computed by fitting an exponential curve and the slope  $m$  is found to be equal to 0.02086264, while the constant  $C$  is equal to  $0.1248 \times 10^{-22}$ , i.e.,

$$P(X) = (0.1248) (10^{-22}) e^{0.02086264X} \quad (E.83)$$

The mean of daily peaks  $\bar{Y}$  is 2011.452 MW and  $\sigma_y$  is equal to 168.1875 while the available capacity is 2560 MW. The computation will be carried out as follows:

1. Compute the limits of the normal distribution truncating at  $2\sigma$ , i.e.,

$$b = \bar{Y} + 2\sigma_Y = 2347.827 \text{ MW}$$

$$a = \bar{Y} - 2\sigma_Y = 1675.077 \text{ MW}$$

2. Compute  $K_1$  and  $k_2$  as follows:

$$k_1 = b - \frac{1}{m}$$

$$= 2347.827 - 47.933 = 2299.894 \quad \text{and}$$

$$k_2 = a - \frac{1}{m}$$

$$= 1675.077 - 47.933 = 1627.144$$

3. The cumulative probability for the available capacity equal to  $b$  is given as

$$\begin{aligned} P(X=b) &= ce^{mb} \\ &= 0.01272807 \end{aligned}$$

also

$$\begin{aligned} P(X=a) &= ce^{ma} \\ &= 1.2 \times 10^{-6} \end{aligned}$$

4. Compute  $\bar{X}$  as follows:

$$\begin{aligned} \bar{X} &= \frac{(2299.894)(.01272807) - (1627.144)(1.2)(10^{-6})}{0.01272207} \\ &= \frac{29.2640 - .001975}{.01272207} = 2300 \end{aligned}$$

$$\text{and } P(\bar{X}) = 0.01272$$

i.e., the risk index for that month is 0.01272 and the Pool requires a new unit or units to be added to the system.

Another method is to assume the month consists of 21 working days. The most likely peak load for one day of the month will be the monthly mean  $\bar{Y}$ . This load will correspond to the peak for one day and will have a value corresponding to the

center of the normal curve as shown in Fig. E.7.

For a second day, the most likely load will include the next 1/21 of the area under the curve (59), with the abscissa equal to  $\bar{Y} + 0.119\sigma_Y$ . Also for the third day, the abscissa will be  $\bar{Y} - 0.119\sigma_Y$ . For the rest of the days similar expressions are given and the peak load will be given as  $\bar{Y} + 1.98\sigma_Y$ . Now, using these 21 values of loads, we use equation 20 to obtain the probabilities of these 21 days, i.e.,

$$P(\bar{Y} + l\sigma_Y) = ce^{m(\bar{Y} + l\sigma_Y)} \quad (\text{E.84})$$

where  $l$  is a constant value that gives the abscissa. Now the average availability  $P(\bar{X})$  for that month will be

$$P(\bar{X}) = \frac{\sum P_i(\bar{Y} + l_i\sigma_Y)}{21}, \quad i = 1, 21 \quad (\text{E.85})$$

For the year we will have 12 values like this and the annual risk or the annual availability will be

$$\text{Annual risk} = \sum_{i=1}^{12} \frac{P(X_i)}{12} \quad (\text{E.86})$$

and this annual risk will be compared to the predetermined planning risk level to decide whether a new unit should be added to the system or not. It is also evident that this method is very easy to compute and there is no need to fit an exponential curve for the cumulative probability for the available capacity. It could be read directly from the table if we round off to the 21 loads in the month.



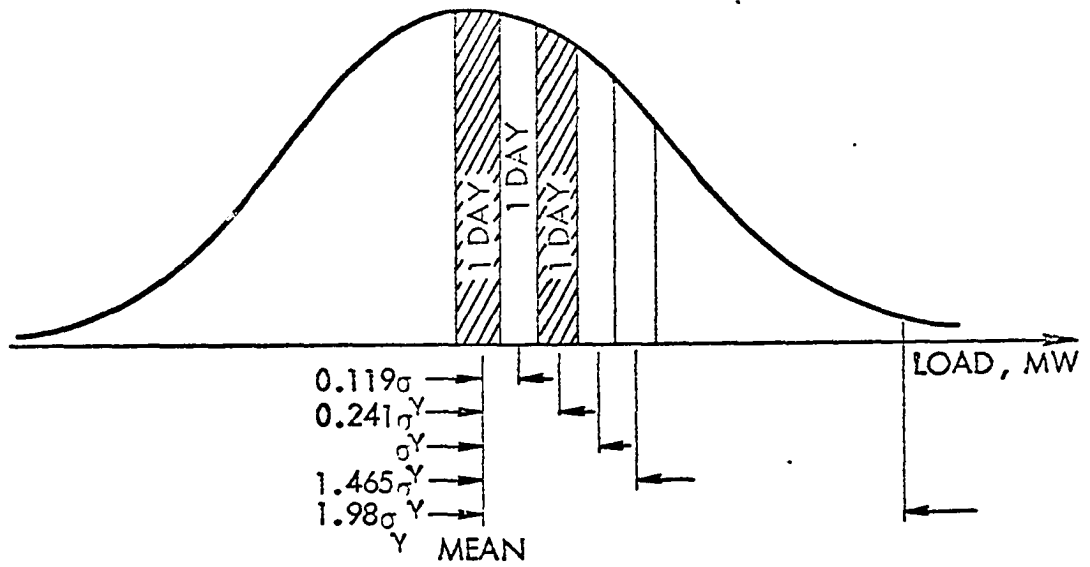


Fig. E.7. Probable deviation of daily loads through a 21-day month

### The Loss of Energy Probability (LOEP)

A second alternative measure of the index of reliability is the loss of energy probability (LOEP). This is done by measuring the expected per unit energy curtailment. It is similar to the LOLP method except that the outage area under the monthly load duration curve is calculated for each outage condition. The energy curtailment on a per unit basis is computed by dividing the probability of the outage by the total area under the load duration curve for that month. The various outage conditions are summed to get the expected per unit energy curtailment which will be a measure of the monthly index of reliability (60). The following example will illustrate this method.

Consider again the system of 4 units each of 20 MW and 3 units of 30 MW each. The total installed capacity will be 170 MW. Assume for a certain month the load duration curve is that shown in Fig. E.8. The peak for that month is 150 MW with a net reserve of 20 MW. Assume that 10 MW of excess reserve be considered. That means the load is 140 MW. The area A is calculated and is found to be 360 MWhr. The total area under the load duration curve is 72000 MWhrs assuming the month is of 30 days. Also, for a load of 130 MW, the area A+B will give the energy curtailment for an outage equal to 40 MW or load equal to 130 MW. This area is found to be 1440 MWhrs. Now the energy loss expected for an outage of 40 MW will be equal to the product of the probability of an outage of 40 MW

given at Table E.4 and the energy curtailment corresponding to that outage.

$$\begin{aligned} \text{i.e., Energy Loss for 40 MW outage} &= 0.002168 \times 1440 \times 1000 \\ &= 3121.920000 \text{ KWHRs.} \end{aligned}$$

These computations are given in the following table.

Table E.10. Loss of energy computation

Forced Outage MW	Probability of forced outage (1)	Amount reserve is exceeded, MW	Peaking energy Kw-Hr (2)	Expected energy loss (1) x (2) Kw-Hr
0	0.868125	0	0	0
10	-----	0	0	0
20	0.070868	0	0	0
30	0.053150	10	360,000	19,134.00
40	0.002168	20	1,440,000	3,121.92
50	0.004339	30	3,240,000	14,058.36
60	0.001115	40	5,760,000	6,422.40
70	0.001133	50	9,000,000	10,197.00
80	0.000089	60	12,960,000	1,153.44
90	0.000002	70	17,640,000	35.28
100	0.000003	80	23,040,000	69.12
110	-----	90	29,160,000	0.00
120	-----	100	36,000,000	0.00
Total expected energy loss by forced outages				54,191.52

System energy available for the month = 72,000,000

Kw-Hr = Area under the load duration curve given in Fig. E.8 x 720

$$\begin{aligned} \text{Probability of energy loss} &= \frac{\text{Expected energy loss}}{\text{System energy available}} \\ &= \frac{54,191.52}{72,000,000.00} = 0.00075266 \end{aligned}$$

i.e., for every 10,000 days, we will have 8 days in which the system energy will be less than the required energy for the month, or for every 10 years, we will have 2 days or every 5 years we will have 1 day for which we have insufficient energy to supply the loads.

The index of reliability (R) will be equal to

$$R = 1.0 - .00075 = 0.99925 \quad (\text{E.87})$$

The calculations above are for one month. Different months will be treated the same way except that the peak load demands are different and the dependable capacity may also change.

To compute an annual index of reliability, we have to divide the sum of the monthly Kw-Hr after subtracting the expected energy losses due to forced outages by the estimated total available energy in Kw-Hr for the whole year.

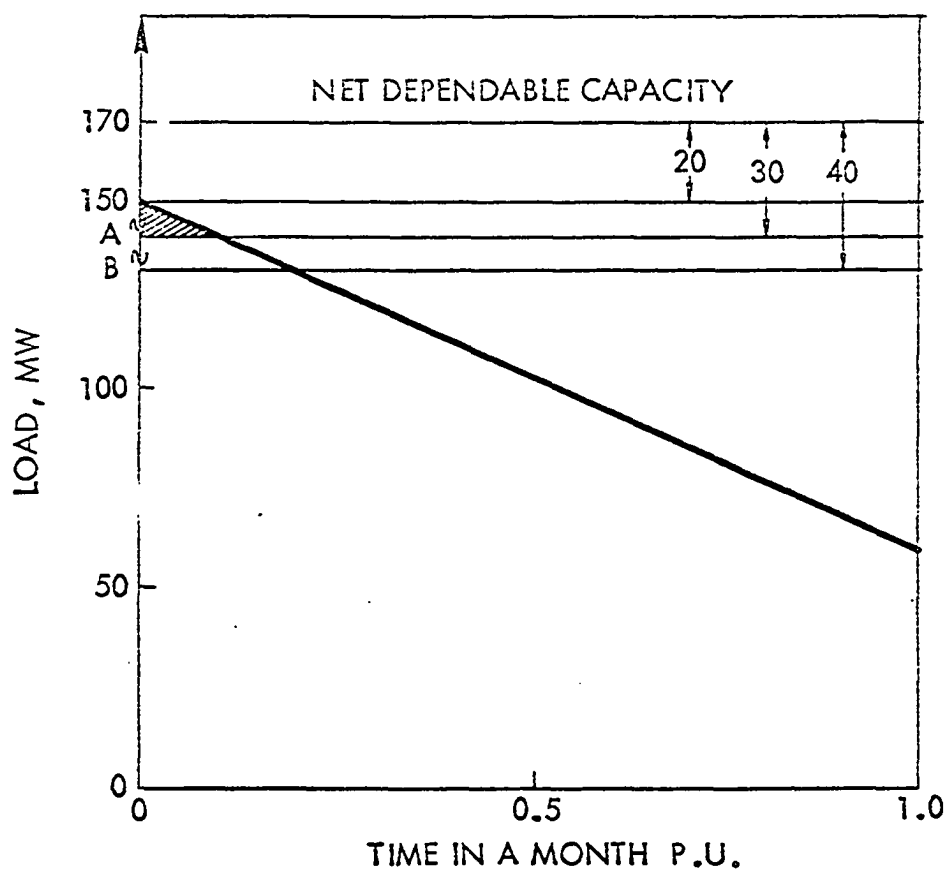


Fig. E.8. Load duration curve

### The Loss of Capacity Probability (LOCP)

This method not only measures the reliability of a power system, but also determines the required reserve capacity on the basis of the projected twelve monthly peak loads.

The expectation of forced outages are calculated by probability methods. This gives results in terms of frequencies, intervals and average durations (61). When we say that a generator has a forced outage rate or a probability of failure equal to 0.02, we mean that every 200 days we have 4 days of failure, or for every 100 days, we have 2 days of failure and so on. But there must be a unique understanding of the definition of forced outage rate.

Assume a unit of 20MW capability has a series of forced outages expressed in terms of the outage duration in days and the interval between individual outages in the same units. For example, suppose the historical record for the unit is that shown in Table. E.13.

Table E.11. Historical record for the unit

Outage duration in days (t)	3	1.5	1.5	2
Interval between outages in days (T)	125	95	75	105

The average duration of individual outages in days will be  
 $t = (3+1.5+1.5+2)/4 = 8/4 = 2$  days.

The average interval of individual outages in days will be given also as

$$T = (125+95+75+105)/4 = 400/4 = 100 \text{ days.}$$

We can then say that the forced outage rate is

$$p = \frac{t}{T} = \frac{2}{100} = 0.02 \quad (\text{E.88})$$

The frequency of individual events is

$$F = \frac{1}{T} = \frac{p}{t} = \frac{1}{100} = 0.01 \quad (\text{E.89})$$

Suppose we assume that the unit is in service with probability 0.98. Since the interval between the (outage) events, when the unit is in service, will be the same as the interval the unit is out of service, we may write

$$T = \frac{t_{in}}{(1-p)} = \frac{t_{out}}{p} \quad (\text{E.90})$$

where  $t_{in}$  is the duration of the in-service period and  $t_{out}$  is the duration of the out-of-service period.

Now, applying this equation and substituting  $q = 1 - p$ , we have

$$t_{in} = t_{out} \cdot \frac{q}{p} \quad (\text{E.91})$$

and

$$F_{in} = \frac{q}{t_{in}} \quad (\text{E.92})$$

These computations are summarized in Table. E.12. for our example.

Table E.12. Computations for one unit

MW Outage	Probability	Duration in days	Frequency	Interval in days
0	0.98000	98.0	0.01	100.00
20	.02	2.0	0.01	100.00

Now add a second unit of the same size with the same forced outage rate in two steps as before.

First, consider the unit is in service with 0 MW outage. The duration of this event will be given as

$$t_{12} = \frac{t_1 t_2}{t_1 + t_2} \quad (\text{E.93})$$

where  $t_{12}$  is the duration of having both units in service at the same time,  $t_1$  and  $t_2$  are the individual durations that unit one and unit two will be on service. Also,

$$P_{12} = P_1 \cdot P_2 \quad (\text{E.94})$$

Then

$$F_{12} = \frac{P_{12}}{t_{12}} \quad (\text{E.95})$$

$$T_{12} = \frac{1}{F_{12}} \quad (\text{E.96})$$

After some computation, we will get the following results for outage equal 0.

$$p(0) = 0.98 \times .98 = 0.960399$$

$$t(0) = \frac{98 \times 98}{2(98)} = 49.00 \text{ days.}$$



$$F(0) = \frac{.960399}{49} = 0.0196 \quad \text{and}$$

$$T(0) = \frac{1}{0.0196} = 51.0204 \text{ days.}$$

The following will give the summary of the first steps.

<u>MW Outage</u>	<u>Probability</u>	<u>Duration</u>	<u>Frequency</u>	<u>Intervals</u>
0 + 0 = 0	0.960399	49.000	0.019600	51.0204
20 + 0 = 20	0.0196	1.9216	0.0102	98.0392

The second step is to add the unit with its entire capacity as an outage with  $p = 0.02$  and apply equations E.94 through E.96 with the result:

<u>MW Outage</u>	<u>Probability</u>	<u>Duration</u>	<u>Frequency</u>	<u>Intervals</u>
0 + 20 = 20	0.0196	1.9600	0.01	100.00
20 + 20 = 40	0.0004	0.9899	0.000408	2474.749

Now we can combine the two steps. The only stumbling block is that we have two values of 20 MW outage. To handle this situation, we perform the following computations.

$$1. \quad p(20) = p_1(20) + p_2(20) \text{ where 1 \& 2 refer to steps 1 and 2 respectively.}$$

$$2. \quad F(20) = F_1(20) + F_2(20) \quad (\text{E.97})$$

$$3. \quad T(20) = \frac{1}{F(20)} \quad (\text{E.98})$$

$$4. \quad t(20) = \frac{p(20)}{F(20)} \quad (\text{E.99})$$

Applying these four equations, we have

$$p(20) = 0.0196 + 0.0196 = 0.03920$$

$$F(20) = 0.0102 + .01 = 0.0202$$

$$T(20) = \frac{1.0}{0.0202} = 49.5049$$

$$t(20) = \frac{0.0392}{0.0202} = 1.9600$$

Now we will have the combined system as shown in Table E.13.

Table E.13. Loss of capacity probability computations for two units

MW Outage	Probability	Duration	Frequency	Interval
0	0.960399	49.0000000	0.019600	51.0204
20	0.039200	1.9600000	0.020200	49.5049
40	0.000400	0.0000003	0.000404	2474.7490

We now add the next two units following the same steps to find the following results.

Table E.14. Loss of capacity probability computation for four units

MW Outage	Probability	Duration	Frequency	Interval
0	0.92236810	24.5000	0.03764770	26.5620
20	0.07529533	1.8608	0.04052888	24.6738
40	0.00230496	0.9671	0.00238728	418.8867
60	0.00003136	0.6533	0.00004808	20798.6800
80	0.00000016	0.0000	0.00000320	307788.0000

From the above calculation, we can see that this method could be used to test the reliability of the system.

Now we consider the problem of computing the spinning reserve requirement. If  $Y_m$  is the monthly peak for any month under study and  $I$  is the installed capacity, how can we calculate the spinning reserve such that the reliability index should be equal to the selected level? Let  $Y_m = 60$  MW and  $I = 80$  MW. From the interval column in Table E.14 before it is clear that at 418.8 days or 1.7 years we will have outage of magnitude 40 MW. Then our spinning reserve should be not less than 40 MW if we allow a capacity deficiency of one day in 1.7 years.

Now, since  $I = 80$  MW and  $Y_m = 50$ , the spinning reserve will be 20 MW which will cause a capacity deficiency of 1 day in one month and our system is not reliable in the sense that it does not measure up to our criterion of reliability. The reason for this is that if we lose any unit, which could mean a 2-day outage every 100 days, we will lose 20 MW or 25% of the total capacity. Also, we could lose 40 MW or 50% of the capacity every 2 years for 1 day which is much greater than the allowed risk index. We conclude that this system is not well planned to meet such a load. It is obvious that we must select a constant risk index (1 day every 10 or 5 years), and check the spinning reserve at each load level. Should the generation capacity be less than the peak of the load plus the computed spinning reserve at the selected load level, we have to add a new

capacity or purchase through the ties with other systems.

From all the above methods, the last method was selected as the basis for a computer program which was prepared to combine more than 150 units. Also, the Pool study results for January 1970 are shown.

A discussion on the effect of a unit addition on the spinning reserve will be given later in Appendix F. Also, the megawatt outage, the probability of such an outage, the duration, and the interval for the Pool is shown in Table E.15. The cumulative probability of the available capacity is given in Table E.16. The outage probabilities and the intervals in days for the Iowa Pool existing system are plotted versus the MW outages as shown in Fig. E.9. The cumulative probabilities for the same system are plotted versus the MW outages as in Fig. E.10. A flow chart for the program is given in Fig. E.11 while a Fortran list is shown at the end of this appendix for the computer program labelled as Program 5.

Table E.15. The probabilities, durations and intervals of the MW outages for the original Iowa Pool system

MW Outage	Probability	Duration	Interval
0	0.23224300	1.35578900	5.8378
5	0.01421895	0.80802820	56.8276
10	0.05716519	0.80692260	14.1156
15	0.02244278	0.76039150	33.8813
20	0.13084570	0.78934210	6.0326
25	0.04104059	0.71629850	17.4534
30	0.04286018	0.61097730	14.2551
35	0.03429667	0.63832970	18.6120
40	0.05110363	0.59125230	11.5699
45	0.02568068	0.53915320	20.9945
50	0.03945166	0.61091030	15.4850
55	0.01908101	0.50425690	26.4272
60	0.02687450	0.53318900	19.8400
65	0.01275925	0.46105110	36.1346
70	0.02428740	0.54024300	22.2437
75	0.01059601	0.45653930	43.0860
80	0.01719527	0.52342150	30.4398
85	0.00779890	0.44200130	56.6748
90	0.01598127	0.53183910	33.2789
95	0.00626035	0.43157780	68.9383
100	0.00933547	0.46849180	50.1841
105	0.00492924	0.42654830	86.6926
110	0.00757189	0.45938640	60.5700
115	0.00382917	0.41730860	108.9814
120	0.01039071	0.57997950	55.8171
125	0.00307019	0.41325940	134.6040
130	0.00489883	0.45334480	92.5415
135	0.00240981	0.40761470	169.1482

Table E.15 (Cont.)

MW Outage	Probability	Duration	Interval
140	0.01757598	0.66625640	37.9072
145	0.00302669	0.45221540	149.4091
150	0.01158744	0.61200090	52.8159
155	0.00323315	0.46649120	144.2838
160	0.01056696	0.52477870	49.6622
165	0.00392406	0.47352520	120.5721
170	0.00731320	0.49422010	67.5792
175	0.00369714	0.46060850	124.5849
180	0.00486428	0.44086960	90.5342
185	0.00279111	0.42230160	151.3022
190	0.00426706	0.44621840	104.5728
195	0.00208752	0.39612200	189.7569
200	0.00897250	0.62139140	69.2551
205	0.00182265	0.40354420	221.4050
210	0.00393369	0.46417350	117.9996
215	0.00167904	0.41217660	245.4830
220	0.00517759	0.49793080	96.1703
225	0.00189592	0.43618650	230.0664
230	0.00260115	0.43073150	165.5928
235	0.00152713	0.41738380	273.3125
240	0.00234372	0.42312110	180.5341
245	0.00115882	0.38701630	333.9744
250	0.00174464	0.42053890	241.0465
255	0.00086992	0.37311530	428.9080
260	0.00148654	0.41783230	281.0769
265	0.00062155	0.35581940	572.4753
270	0.00118774	0.41158810	346.5303
275	0.00049956	0.35342070	707.4641
280	0.00104134	0.41455420	398.0974

Table E.15 (Cont.)

MW Outage	Probability	Duration	Interval
285	0.00040634	0.35223920	866.8628
290	0.00105798	0.43503910	411.1992
295	0.00035256	0.35335580	1002.2650
300	0.00060542	0.37980860	627.3506
305	0.00029817	0.35153540	1178.9910
310	0.00055467	0.38228290	689.2043
315	0.00025710	0.34972440	1360.2910
320	0.00046542	0.40115640	861.9231
325	0.00019713	0.33948040	1722.1040
330	0.00029937	0.35764180	1194.6540
335	0.00014847	0.32791030	2208.5960
340	0.00057849	0.45553270	787.4553
345	0.00014002	0.34067730	2432.9770
350	0.00039316	0.42757150	1087.5320
355	0.00012744	0.34729690	2725.2290
360	0.00034501	0.39474680	1144.1680
365	0.00013496	0.35740350	2648.1910
370	0.00024165	0.37833670	1565.6320
375	0.00012038	0.35407660	2941.4460
380	0.00016580	0.35087470	2116.2480
385	0.00009095	0.33447260	3677.4490
390	0.00013595	0.35150730	2585.5610
395	0.00006785	0.32021830	4719.7380
400	0.00010058	0.34327700	3413.0590
405	0.00004798	0.30801320	6419.4370
410	0.00008446	0.34236980	4053.8060
415	0.00003626	0.30157560	8316.8980
420	0.00005977	0.33087310	5536.1520
425	0.00002788	0.29863350	10711.6100

Table E.15 (Cont.)

MW Outage	Probability	Duration	Interval
430	0.00005359	0.33772990	6302.4880
435	0.00002205	0.29611080	13428.1000
440	0.00003381	0.31772380	9397.4050
445	0.00001698	0.29227250	17210.7300
450	0.00002609	0.31008580	11885.5800
455	0.00001313	0.28859230	21977.5500
460	0.00002473	0.33316990	13471.9500
465	0.00000998	0.28492150	28551.5700
470	0.00001778	0.32162200	18092.2500
475	0.00000754	0.28101000	37266.1800
480	0.00001790	0.33392890	18656.0700
485	0.00000653	0.28637380	43861.1300
490	0.00001837	0.35378460	19257.3400
495	0.00000578	0.29180710	50500.0600
500	0.00001078	0.31363080	29088.9100
505	0.00000505	0.29229520	57836.7800
510	0.00001016	0.31866620	31375.3700
515	0.00000454	0.29387680	64680.5600
520	0.00000581	0.29461930	50690.5000
525	0.00000338	0.28409650	83981.5000
530	0.00000489	0.29346450	60029.9000
535	0.00000247	0.27288780	110523.9000
540	0.00000360	0.29176440	81057.1200
545	0.00000177	0.26618050	150133.5000
550	0.00000277	0.28678400	103636.1000
555	0.00000127	0.25940590	205059.3000
560	0.00000207	0.28265910	136694.5000
565	0.00000097	0.25782760	266250.3000
570	0.00000154	0.27902990	181069.2000



Table E.15 (Cont.)

MW Outage	Probability	Duration	Interval
575	0.00000073	0.25522600	351435.2000
580	0.00000114	0.27516830	242116.0000
585	0.00000055	0.25221280	461095.0000
590	0.00000074	0.26307880	353528.6000
595	0.00000040	0.24876370	618859.8000
600	0.00000064	0.27186100	424878.9000
605	0.00000030	0.24570440	831237.8000
610	0.00000057	0.28417250	497203.3000
615	0.00000022	0.24387200	1109125.0000
620	0.00000036	0.26323780	739642.3000
625	0.00000017	0.24438750	1426163.0000
630	0.00000040	0.28572020	713045.3000
635	0.00000015	0.24920350	1694434.0000
640	0.00000020	0.25612530	1288481.0000
645	0.00000011	0.24622800	2202937.0000
650	0.00000019	0.26422390	1363581.0000
655	0.00000009	0.24501480	2717069.0000
660	0.00000011	0.25219930	2198472.0000
665	0.00000006	0.24026030	3697334.0000
670	0.00000009	0.24765120	2836673.0000
675	0.00000005	0.23283550	5137201.0000
680	0.00000007	0.24919370	3761093.0000
685	0.00000003	0.23001970	6991407.0000
690	0.00000005	0.24398560	5274665.0000
695	0.00000002	0.22520200	9934541.0000
700	0.00000004	0.24477350	6670723.0000
705	0.00000002	0.22415710	0.1319E 08
710	0.00000002	0.23749630	9901863.0000
715	0.00000001	0.22253540	0.1802E 08

Table E.15 (Cont.)

MW Outage	Probability	Duration	Interval
720	0.00000002	0.23763410	0.1278E 08
725	0.00000001	0.22057580	0.2422E 08
730	0.00000001	0.22751080	0.2005E 08
735	0.00000001	0.21765050	0.3369E 08
740	0.00000001	0.22633090	0.2690E 08
745	0.00000000	0.21423640	0.4744E 08
750	0.00000001	0.24287900	0.3073E 08
755	0.00000000	0.21241080	0.6610E 08
760	0.00000000	0.22379380	0.5098E 08
765	0.00000000	0.21081790	0.9223E 08
770	0.00000000	0.23108760	0.5835E 08
775	0.00000000	0.21411860	0.1171E 09
780	0.00000000	0.21762490	0.1008E 09
785	0.00000000	0.21153590	0.1635E 09
790	0.00000000	0.21696430	0.1317E 09
795	0.00000000	0.20676940	0.2296E 09
800	0.00000000	0.21757660	0.1878E 09
805	0.00000000	0.20383500	0.3296E 09
810	0.00000000	0.21008840	0.2850E 09
815	0.00000000	0.19920090	0.4914E 09
820	0.00000000	0.21180240	0.3785E 09
825	0.00000000	0.19830360	0.6901E 09
830	0.00000000	0.20922760	0.5677E 09
835	0.00000000	0.19600540	0.1013E 10
840	0.00000000	0.21192620	0.7311E 09
845	0.00000000	0.19467920	0.1428E 10
850	0.00000000	0.20369190	0.1168E 10
855	0.00000000	0.19426160	0.2013E 10
860	0.00000000	0.20272860	0.1612E 10

Table E.15 (Cont.)

MW Outage	Probability	Duration	Interval
865	0.00000000	0.19330230	0.2832E 10
870	0.00000000	0.19645780	0.2594E 10

Table E.16. The cumulative probabilities of the available capacities of the original Iowa Pool system

Index	Available capacity	Cumulative probability
1	2560.00000000	0.99998710
2	2555.00000000	0.76774420
3	2550.00000000	0.75352530
4	2545.00000000	0.69535010
5	2540.00000000	0.67391730
6	2535.00000000	0.54307160
7	2530.00000000	0.50203110
8	2525.00000000	0.45917090
9	2520.00000000	0.42487430
10	2515.00000000	0.37377070
11	2510.00000000	0.34809000
12	2505.00000000	0.30863850
13	2500.00000000	0.28955750
14	2495.00000000	0.26268300
15	2490.00000000	0.24992380
16	2485.00000000	0.22563540
17	2480.00000000	0.21504050
18	2475.00000000	0.19784530
19	2470.00000000	0.19004640
20	2465.00000000	0.17406520
21	2460.00000000	0.16780490
22	2455.00000000	0.15846950
23	2450.00000000	0.15354930
24	2445.00000000	0.14597740
25	2440.00000000	0.14214820
26	2435.00000000	0.13175760
27	2430.00000000	0.12868740
28	2425.00000000	0.12378860
29	2420.00000000	0.12137880

Table E.16 (Cont.)

Index	Available capacity	Cumulative probability
30	2415.00000000	0.10380290
31	2410.00000000	0.10077630
32	2405.00000000	0.08918893
33	2400.00000000	0.08595580
34	2395.00000000	0.07538903
35	2390.00000000	0.07146508
36	2385.00000000	0.06415230
37	2380.00000000	0.06045737
38	2375.00000000	0.05559309
39	2370.00000000	0.05280198
40	2365.00000000	0.04853492
41	2360.00000000	0.04644740
42	2355.00000000	0.03747489
43	2350.00000000	0.03565224
44	2345.00000000	0.03171856
45	2340.00000000	0.03003952
46	2335.00000000	0.02486192
47	2330.00000000	0.02296601
48	2325.00000000	0.02036486
49	2320.00000000	0.01883774
50	2315.00000000	0.01649402
51	2310.00000000	0.01533520
52	2305.00000000	0.01359056
53	2300.00000000	0.01272065
54	2295.00000000	0.01123411
55	2290.00000000	0.01061257
56	2285.00000000	0.00942483
57	2280.00000000	0.00892527
58	2275.00000000	0.00788394

Table E.16 (Cont.)

Index	Available capacity	Cumulative probability
59	2270.00000000	0.00747760
60	2265.00000000	0.00641964
61	2260.00000000	0.00505708
62	2255.00000000	0.00546167
63	2250.00000000	0.00516351
64	2245.00000000	0.00450884
65	2240.00000000	0.00435176
66	2235.00000000	0.00388647
67	2230.00000000	0.00368934
68	2225.00000000	0.00338997
69	2220.00000000	0.00324150
70	2215.00000000	0.00266302
71	2210.00000000	0.00252299
72	2205.00000000	0.00212983
73	2200.00000000	0.00200240
74	2195.00000000	0.00165739
75	2190.00000000	0.00152243
76	2185.00000000	0.00128078
77	2180.00000000	0.00116040
78	2175.00000000	0.00099460
79	2170.00000000	0.00090365
80	2165.00000000	0.00076770
81	2160.00000000	0.00059985
82	2155.00000000	0.00059928
83	2150.00000000	0.00055129
84	2145.00000000	0.00046684
85	2140.00000000	0.00043058
86	2135.00000000	0.00037081
87	2130.00000000	0.00034293

Table E.16 (Cont.)

Index	Available capacity	Cumulative probability
88	2125.00000000	0.00028935
89	2120.00000000	0.00026730
90	2115.00000000	0.00023349
91	2110.00000000	0.00021651
92	2105.00000000	0.00019042
93	2100.00000000	0.00017729
94	2095.00000000	0.00015256
95	2090.00000000	0.00014258
96	2085.00000000	0.00012480
97	2080.00000000	0.00011726
98	2075.00000000	0.00009936
99	2070.00000000	0.00009284
100	2065.00000000	0.00007446
101	2060.00000000	0.00006869
102	2055.00000000	0.00005790
103	2050.00000000	0.00005285
104	2045.00000000	0.00004269
105	2040.00000000	0.00003815
106	2035.00000000	0.00003234
107	2030.00000000	0.00002896
108	2025.00000000	0.00002407
109	2020.00000000	0.00002160
110	2015.00000000	0.00001800
111	2010.00000000	0.00001623
112	2005.00000000	0.00001346
113	2000.00000000	0.00001219
114	1995.00000000	0.00001013
115	1990.00000000	0.00000916
116	1985.00000000	0.00000762

Table E.16 (Cont.)

---

Index	Available capacity	Cumulative probability
117	1980.00000000	0.00000689
118	1975.00000000	0.00000575
119	1970.00000000	0.00000521
120	1965.00000000	0.00000446
121	1960.00000000	0.00000406
122	1955.00000000	0.00000342
123	1950.00000000	0.00000312
124	1945.00000000	0.00000255
125	1940.00000000	0.00000233
126	1935.00000000	0.00000198
127	1930.00000000	0.00000181
128	1925.00000000	0.00000141
129	1920.00000000	0.00000126
130	1915.00000000	0.00000106

---



Fig. E.9. The outage probabilities and the intervals in days versus the MW outages for the Iowa Pool original system

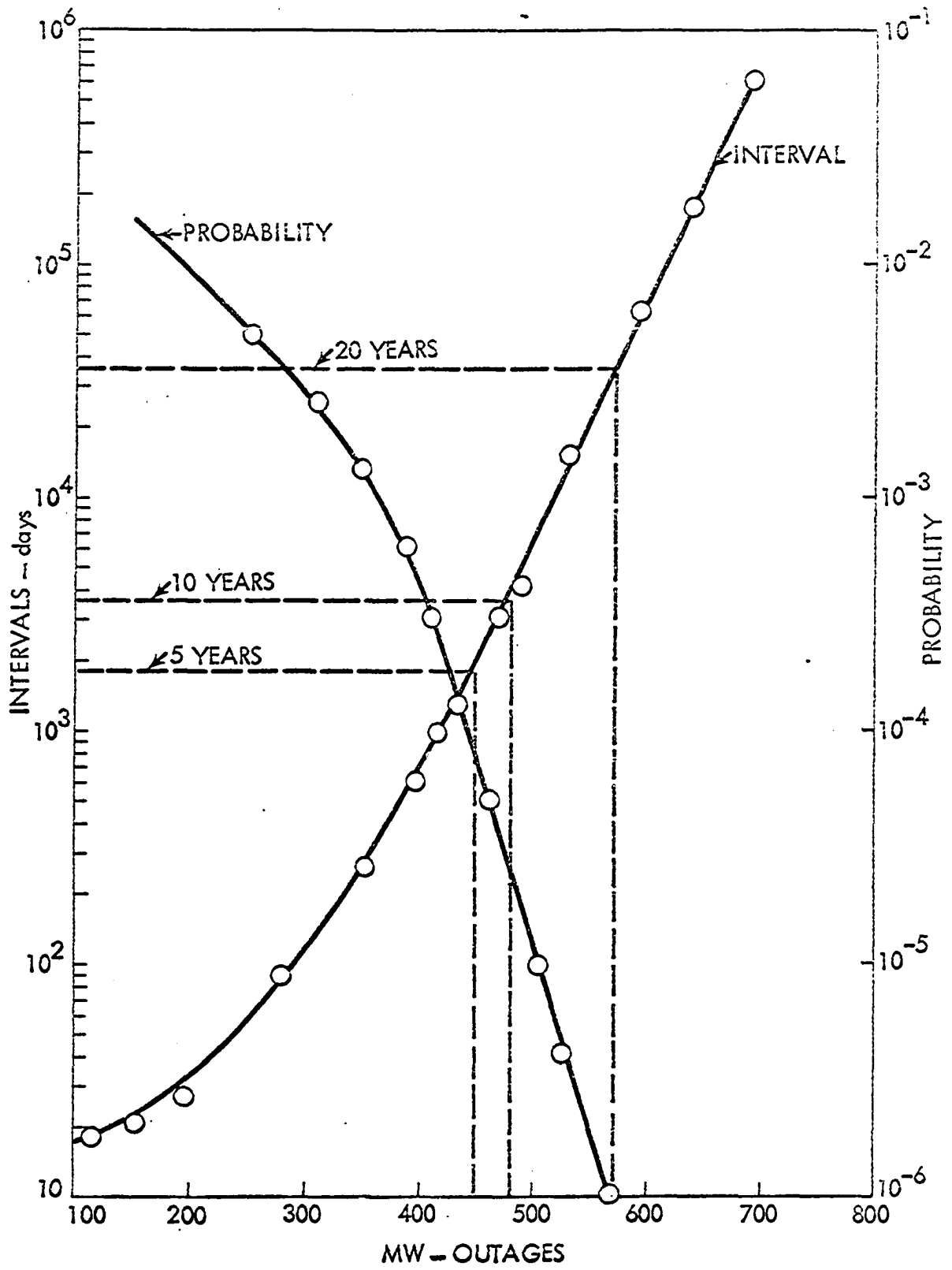
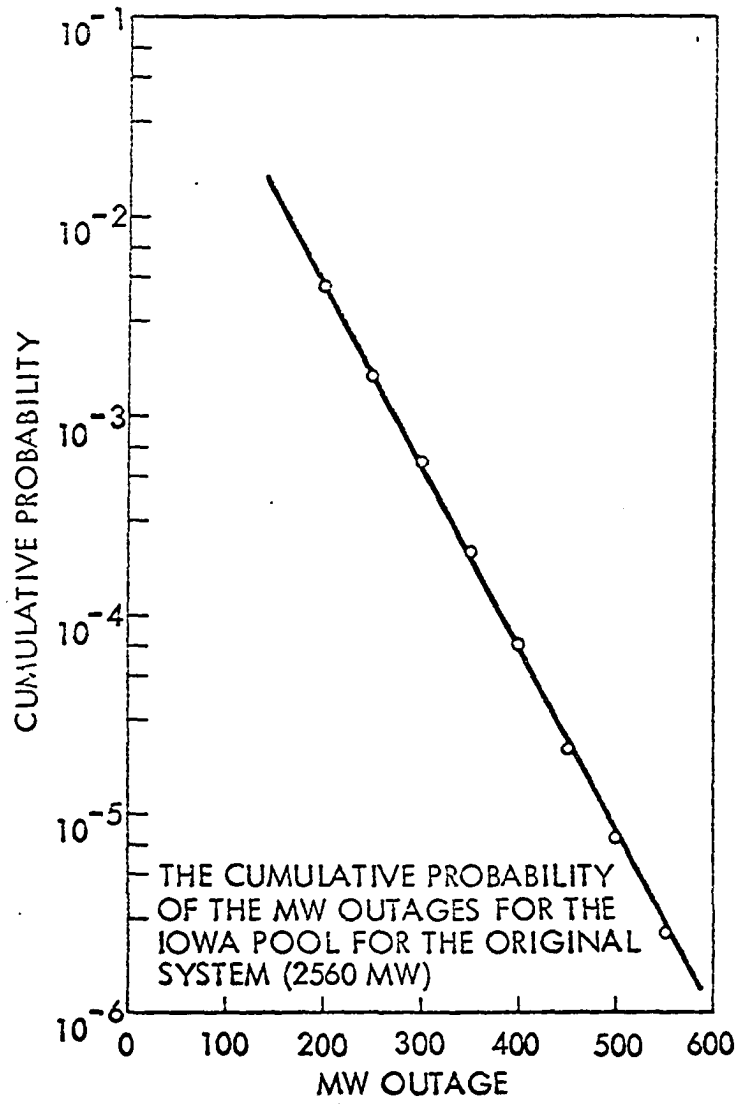


Fig. E.10. The cumulative probability of the MW outages  
for the Iowa Pool for the original system  
(2560 MW)



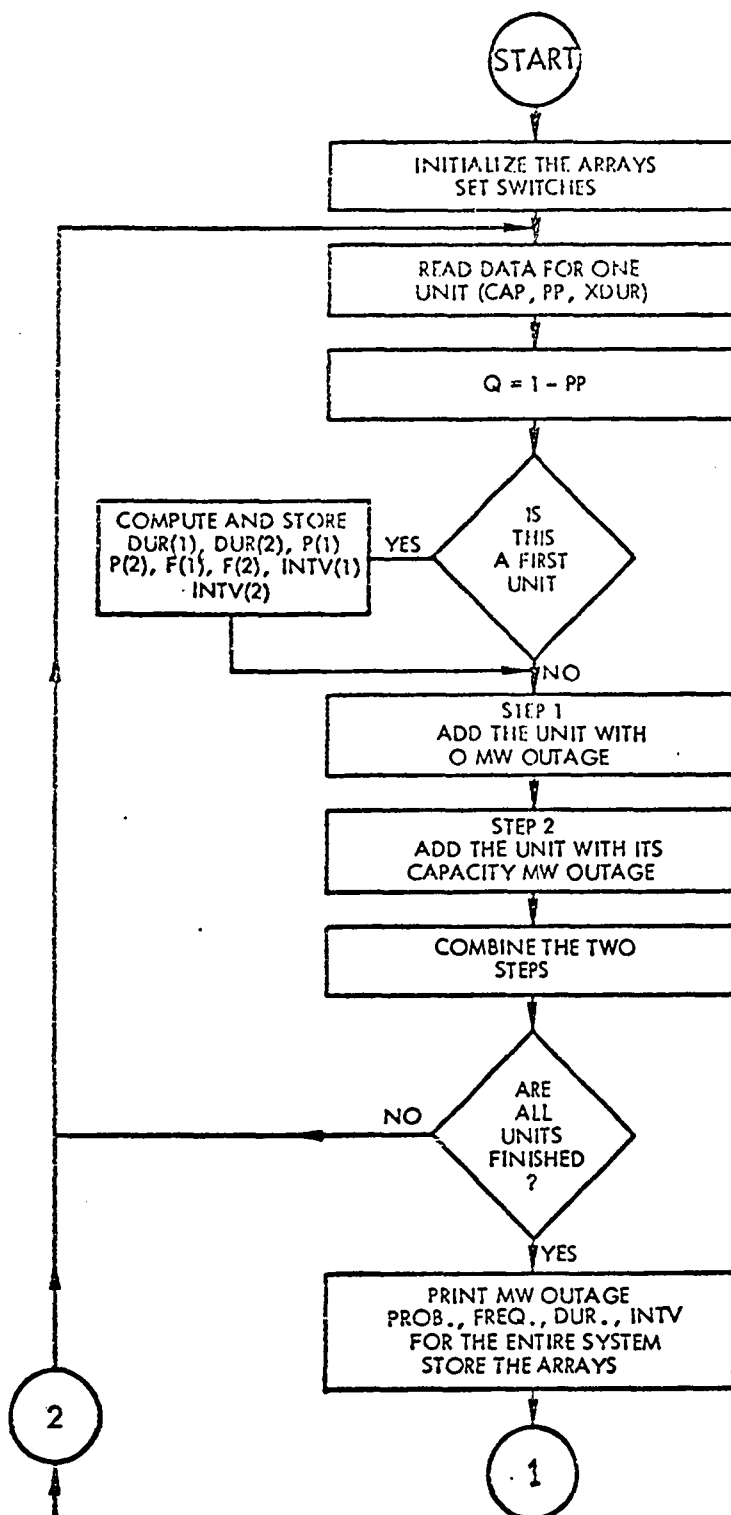


Fig. E.11. Subroutine risk

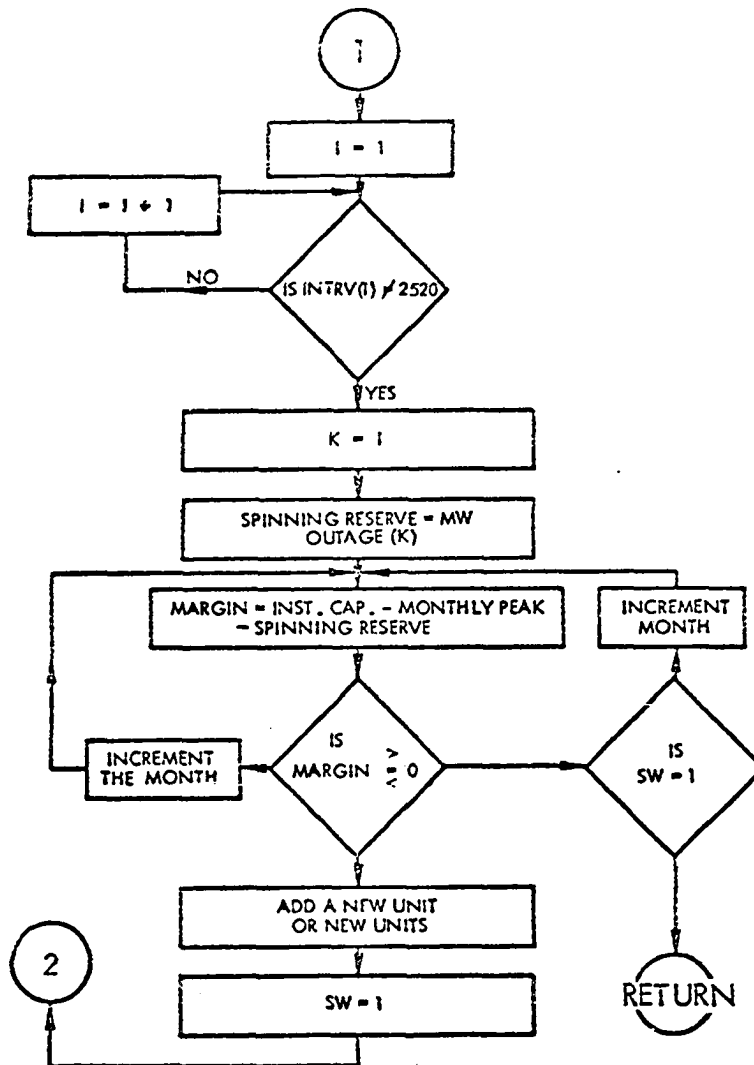


Fig. E.11 (Cont.)

```

C      IMPLICIT INTEGER*2(0)
C      *****
C      PROGRAM TO MEASURE THE SYSTEM RELIABILITY, COMPUTE THE PRODUCTION COST,
C      ADD A NEW UNIT, COMPUTE TOTAL COST OF THE NEW ADDITION AND THE PRESENT
C      WORTH COST OF THE EXPANSION PATTERNS
C      *****
C      DIMENSION X(20)
C      DIMENSION COFSIG(10), SIGVRG(12)
C      DIMENSION PWCOST(80)
C      DIMENSION AINVRT(12)
C      DIMENSION AY(60)
C      DIMENSION DDD(100), EEE(100), CY(100), COSMP(100)
C      DIMENSION NEWUNT(20), YY1(12), YY2(12)
C      DIMENSION  INVEN3(3), PMNUC2(3), INVEN4(3)
C      DIMENSION  IHEATN(3), INVEN1(3), INVEN2(3), TOTNC2(3)
C      DIMENSION  DOLNU2(3), PMNUC1(3), TOTNC1(3), IMPRNC(3), MTPRNC(3)
C      DIMENSION ENPURC(5), ENSOLD(5), NEWCAP(5)
C      DIMENSION SIZE(240), IHEATR(5), SNUCL(3), DOLNC1(5)
C      DIMENSION SIZEKW(100), DOLKW1(100), DOLKW2(100), TOTKW1(100)
C      DIMENSION TOTKW2(100), PMKW1(100), PMKW2(100), SLP(20), C(20)
C      DIMENSION ANFUEL(100), IRET(100), OCAPC(100), JP(100)
C      DIMENSION SLOPE(400), AVPEKB(300), FACTLD(100)
C      DIMENSION RISK(300), PEAK(300)
C      DIMENSION OCM(1500), OCM1(1500), P(1500), P1(1500), CM(1500)
C      DIMENSION F(1500), FF(1500), DUR(1500), DURR(1500), AINTV(1500)
C      DIMENSION F2(2), DUR2(2), P2(2)
C      FUNP(GG, HH, W) = GG * EXP(HH * W)
C      FUNC(Z) = 1.0 / EXP((Z * Z) / 2.0)
C      INITIALIZING DATA ARRAYS
C      DATA RISKYR, CI, AVGOUT, SIGMLS, CCI, TOTPW/0., 0., 0., 0., 0., 0., 0., AA/0./
C      DATA IHEATR /'9500', '9400', '9400', '9200', '9200'/
C      DATA IMPRNC /'450', '575', '575'/
C      DATA MTPRNC /'300', '450', '450'/
C      DATA INVEN1 /'175', '255', '345'/
C      DATA INVEN2 /'175', '255', '345'/
C      DATA INVEN3 /'160', '235', '320'/

```

Program 5. Reliability and cost program

```

DATA INVEN4 /'145','215','295'/
DATA CCY3,PURCH,SALCST/0.0,0.0,0.0/
N=9
YR=71
YZ=70.0
SUMFUL=0.0
PWY=70.0
AINR=0.07
FB=.03
JR=0
TOTCST=0.0
JSW=0
KKL=0
RISKY=0.0
KSW=0
LL=0
TMAX=1.0E+21
KS=2
I=0
CAPURC=0.0
COSTMP=0.0
LP=0
KSL=0
IM=0
NEWS=0
KPR=0
JJS=0
IC=0
JB=50
IRT=71
FIXC=1.125
KK = 0
C *****
C READ DATA
C *****
READ(1,8885) (COFSIG(I),I=1,10)
Program 5 (Cont.)

```



```

8885 FORMAT(5F10.4)
      READ(1,8886) (SIGVRG(I),I=1,12)
8886 FORMAT(6F10.7)
      READ(1,8881) (SIZEKW(I),I=1,5)
8881 FORMAT(5F10.1)
      READ(1,8882) (DOLKW1(I),I=1,5)
      READ(1,8882) (DOLKW2(I),I=1,5)
      READ(1,8882) (PMKW1(I),I=1,5)
      READ(1,8882) (PMKW2(I),I=1,5)
      READ(1,8882) (TOTKW1(I),I=1,5)
      READ(1,8882) (TOTKW2(I),I=1,5)
8882 FORMAT(5F10.3)
      READ(1,8883) (PMNUC1(I),I=1,3)
      READ(1,8883) (PMNUC2(I),I=1,3)
      READ(1,8883) (TOTNC1(I),I=1,3)
      READ(1,8883) (TOTNC2(I),I=1,3)
      READ(1,8883) (SNUCL(I),I=1,3)
8883 FORMAT(3F10.3)
      READ(1,8884) (IHEATN(I),I=1,3)
8884 FORMAT(3I10)
C      END OF UNITS DATA*****
      DO 7777 I=1,5
7777 SIZEKW(I)=SIZEKW(I)/1000.
      CALL TREXP(5,SIZEKW,DOLKW1,C,SLP,1)
      CALL TREXP(5,SIZEKW,DOLKW2,C,SLP,2)
      CALL TREXP(5,SIZEKW,TOTKW1,C,SLP,3)
      CALL TREXP(5,SIZEKW,TOTKW2,C,SLP,4)
      CALL TREXP(5,SIZEKW,PMKW1,C,SLP,5)
      CALL TREXP(5,SIZEKW,PMKW2,C,SLP,6)
      DO 6666 I=1,6
      WRITE(3,5555) I,SLP(I),C(I)
5555 FORMAT(' ',20X,I10,10X,F18.9,F18.9)
6666 CONTINUE
1505 READ(1,11) ITYPE
      11 FORMAT(I2)
      I=0
Program 5 (Cont.)

```

```
GO TO(701,702,703,704,705,706),ITYPE
701 READ(1,12) (SLOPE(1),I=3,247)
12 FORMAT(5F12.6)
GO TO 1505
702 READ(1,114) (AY(1),I=1,48)
114 FORMAT(10X,4F12.6)
GO TO 1505
703 READ(1,115) (AINVRT(I),I=1,12)
115 FORMAT(6F10.5)
GO TO 1505
```

```
C
C
C
*****
COMPUTING THE COST OF PURCHING ENERGY
*****
704 READ(1,15,ERR=1185) ID,OCAP,IENRGC,ISKOUT,IPP,MAXCAP
15 FORMAT(6I10)
IF(ID.EQ.999) GO TO 1505
KPR=KPR+1
ENPURC(KPR)=FLOAT(OCAP)*FLOAT(IENRGC)*8760.
CAPURC = CAPURC + FLOAT(OCAP)
PURCHS=PURCHS+ENPURC(KPR)
GO TO 704
*****
COMPUTING THE COST OF ENERGY SOLD
*****
C
C
C
705 READ(1,17,ERR=1185) ID,OCAP,ISALEC,ISHOUT,MAXCAP
17 FORMAT(5I10)
IF(ID.EQ.999) GO TO 1505
KSL=KSL+1
ENSOLD(KSL)=FLOAT(OCAP)*FLOAT(ISALEC)*8760.
CAPSAL=CAPSAL+FLOAT(OCAP)
SALCST=SALCST+ENSOLD(KSL)
GO TO 705
```

```
706 READ(1,10,ERR=1185) ID,OCAP,IBTU,IFUELC,IRETID
10 FORMAT(2I8,8X,2I8,38X,12)
IF(IRETID.GT.70) GO TO 706
WRITE(3,7711) ID,OCAP,IBTU,IFUELC,IRETID
```

Program 5 (Cont.)

```

7711 FORMAT(' ',10X,5(10X,I10))
      IF(ID.EQ.999) GO TO 1313
      KK=KK+1
      WRITE(3,5757)  KK
5757 FORMAT(' ',20X,'NO. OF UNITS ADDEA =',I10)
      VY=OCAP
      AIB=IBTU
      AIF=IFUELC
C      *****
C      COMPUTING THE COST OF FUEL EACH MONTH
C      *****
      ANFUEL(KK)=AIB*AIF*VY*0.073/100000.0
      IRET(KK)=IRETD
C      *****
C      COMPUTING THE ANNUAL COST OF OPERATIONS AND MAINTENANCE
C      *****
      CXX=EXP(SLP(5)*VY)
      WRITE(3,5522) ANFUEL(KK),CXX
5522 FORMAT(' ',10X,2F20.6)
      CY(KK)=C(5)*CXX*VY
      WRITE(3,8585) CY(KK)
8585 FORMAT(' ',10X,'COST OF OPER. AND MAINT.=',F15.8)
      COSTMP=COSTMP+CY(KK)
      WRITE(3,5074) COSTMP
5074 FORMAT(' ',10X,'COST OF OPERATION & MAINT.=',F20.8)
      OCAPC(KK) = OCAP
      JP(KK) =KK
      KO = KK
C      *****
C      COMPUTING THE LOSS OF CAPACITY PROBABILITY(LOCP)
C      AND THE LOSS OF LOAD PROBABILITY(LOLP)
C      *****
1001 I=I+1
      IF(OCAP.GT.100) GO TO 1013
      PP=0.02
      XDUR=2.0

```

Program 5 (Cont.)

```

GO TO 1022
1013 IF(OCAP.GT.250) GO TO 1014
      PP=0.025
      XDUR=2.0
      GO TO 1022
1014 IF(OCAP.GT.340) GO TO 1016
      IF(IM.EQ.1) GO TO 1015
      PP=0.03
      XDUR=3.0
      GO TO 1022
1015 PP=0.45
      XDUR=4.0
      IM=0
      GO TO 1022
1016 IF(OCAP.GT.600) GO TO 1018
      IF(IM.EQ.1) GO TO 1017
      PP=0.03
      XDUR=4.
      GO TO 1022
1017 PP=0.045
      XDUR=5.
      GO TO 1022
1018 IF(OCAP.GT.725) GO TO 1020
      IF(IM.EQ.1) GO TO 1019
      PP=0.0375
      XDUR=5.0
      GO TO 1022
1019 PP=0.0575
      GO TO 1022
1020 PP=0.06
1022 Q=1.0-PP
      CI=CI+VY
      SIGMLS=SIGMLS+PP*Q*VY*VY
      AVGOOT=AVGOOT+PP*VY
      WRITE(3,4458) CI, SIGMLS, AVGOOT
4458 FORMAT(' ',40X,3(10X,F15.8))
Program 5 (Cont.)

```

```

WRITE(3,7781) I
7781 FORMAT(' ',20X,'***** NO. OF UNITS ADDED =',I10,'*****')
T=XDUR/PP
DUR2(1)=XDUR*Q/PP
DUR2(2)=XDUR
IF(I.NE.1) GO TO 522
P(1)=Q
P(2)=PP
DUR(1)=DUR2(1)
DUR(2)=DUR2(2)
F(1)=P(1)/DUR(1)
T1=1.0/F(1)
F(2)=P(2)/DUR(2)
T2=1.0/F(2)
OCM(1)=0
OCM(2)=OCAP
GO TO 706
522 DO 500 K=1,KS
J=K+KS+1
OCM(J)=OCM(K)+OCAP
P3=P(K)
P(K)=P3*Q
IF(P(K).GT.1.0E-38) GO TO 540
P(K)=0.0
F(K)=0.0
AINTV(K)=TMAX
DUR(K)=0.0
GO TO 550
540 DUR3=(DUR2(1)*DUR(K))/(DUR2(1)+DUR(K))
DUR(K)=DUR3
IF(DUR3.EQ.0.0) GO TO 550
F3=P(K)/DUR3
IF(F3.EQ.0.0) GO TO 550
T3=1.0/F3
IF(T3.LT.TMAX) GO TO 560
T3=TMAX

```

Program 5 (Cont.)

```

560 F(K)=F3
    AINTV(K)=1.0/F(K)
550 P(J)=P3*PP
    IF(P(J).GT.1.0E-38) GO TO 570
    P(J)=0.0
    F(J)=0.0
    AINTV(J)=TMAX
    DUR(J)=0.0
    LP=K
    GO TO 377
570 DUR4=(DUR2(2)*DUR(K))/(DUR2(2)+DUR(K))
    DUR(J)=DUR4
    IF(DUR4.EQ.0.0) GO TO 500
    F4=P(J)/DUR4
    IF(F4.EQ.0.0) GO TO 500
    T4=1.0/F4
    IF(T4.LT.TMAX) GO TO 610
    T4=TMAX
610 F(J)=F4
    AINTV(J)=1.0/F4
500 CONTINUE
377 IF(LP.EQ.0) GO TO 779
    KM=LP
    KJ=J
    KF=KS+2
    MX=KS-LP
    DO 444 M=KF,KJ
    LM=M-MX
    OCM(LM)=OCM(M)
    P(LM)=P(M)
    F(LM)=F(M)
444 AINTV(LM)=AINTV(M)
    LP=0
    KS=KM
779 P(KS+1)=9.9
    P(2*KS+2)=9.9

```

Program 5 (Cont.)

```

DUR(KS+1)=0.0
DUR(2*KS+2)=0.0
OCM(KS+1)=0
OCM(2*KS+2)=OCAP
WRITE(3,777) KS
777 FORMAT(' ',20X,' SIZE OF THE ARRAY =',I10)
L=0
II=1
J=KS+2
80 L=L+1
IF(OCM(II)-OCM(J)) 122,66,22
122 IF(P(II).EQ.9.9) GO TO 33
OCM1(L)=OCM(II)
P1(L)=P(II)
DURR(L)=DUR(II)
FF(L)=F(II)
II=II+1
GO TO 80
22 IF(P(II).EQ.9.9) GO TO 33
P1(L)=P(J)
OCM1(L)=OCM(J)
DURR(L)=DUR(J)
FF(L)=F(J)
J=J+1
GO TO 80
66 IF(P(II).EQ.9.9) GO TO 33
P1(L)=P(II)+P(J)
OCM1(L)=OCM(II)
FF(L)=F(II)+F(J)
IF(FF(L).EQ.0.0) GO TO 33
DURR(L)=P1(L)/FF(L)
II=II+1
J=J+1
GO TO 80
33 IF(P(J).EQ.9.9) GO TO 333
OCM1(L)=OCM(J)

```

Program 5 (Cont.)

```

P1(L)=P(J)
FF(L)=F(J)
DURR(L)=DUR(J)
J=J+1
GO TO 33
333 KS=L
WRITE(3,777) KS
DO 800 KL=1,L
OCM(KL)=OCM1(KL)
P(KL)=P1(KL)
F(KL)=FF(KL)
DUR(KL)=DURR(KL)
IF(F(KL).LT.1.0E-38) GO TO 837
AINTV(KL)=1.0/F(KL)
GO TO 800
837 AINTV(KL)=TMAX
800 CONTINUE
IF(KSW.EQ.1) GO TO 1002
GO TO 706
1002 IF(JR.NE.1) GO TO 1077
JPP=KS-3
JF=2
DO 907 K=2,KS,3
IF(K.GT.JPP) GO TO 1044
P1(JF)=P(K)+P(K+1)+P(K+2)
UUU=OCM(K)*P(K)+OCM(K+1)*P(K+1)+OCM(K+2)*P(K+2)
OCM1(JF)=UUU/P1(JF)
FF(JF)=F(K)+F(K+1)+F(K+2)
AINTV(JF)=1.0/FF(JF)
JF=JF+1
907 CONTINUE
1044 KS=JF-1
P(1)=P1(1)
F(1)=FF(1)
AINTV(1)=1.0/F(1)
DO 707 N=2,KS

```

Program 5 (Cont.)



```

      P(N)=P1(N)
      OCM(N)=OCM1(N)
      F(N)=FF(N)
      AINTV(N)=1.0/F(N)
707  CONTINUE
1077 CONTINUE
1313 CONTINUE
      WRITE(3,1099)
1099 FORMAT('1',20X,' NEW CASE ')
      MMJ=0
      DO 1150 K=1,KS
      RR=1.0-F(K)
      MMJ=MMJ+1
      IF(P(K).LT.1.0E-38) GO TO 756
      DUR(K)=P(K)/F(K)
      GO TO 754
756  AINTV(K)=TMAX
754  IF(AINTV(K).GT.1.0E07) GO TO 666
      WRITE(3,555) OCM(K),P(K),DUR(K),AINTV(K),RR
555  FORMAT(' ',I10,2F19.8,F20.4,F19.9)
      GO TO 1151
666  WRITE(3,999) OCM(K),P(K),DUR(K),AINTV(K),RR
999  FORMAT(' ',I10,2F19.8,E20.4,F19.9)
1151 IF(MMJ.LT.50) GO TO 1150
      MMJ=0
      WRITE(3,5590)
5590 FORMAT(1H1)
1150 CONTINUE
      DO 900 K=1,KS
      CMP=0.0
      JI=K
      DO 700 N=JI,KS
700  CMP=CMP+P(N)
900  P(K)=CMP
      N=9
      DO 509 JC=1,KO

```

Program 5 (Cont.)

```

VZ=DCAPC(JC)
IF(IRET(JC).NE.IRT) GO TO 509
CCI=CCI+VZ
509 CONTINUE
CI=CI+200.0
DD 4798 KH=1,KS
VW=OCM(KH)
CM(KH)=CI-VW
WRITE(3,2244) KH,CM(KH),P(KH)
2244 FORMAT(' ',20X,I10,10X,2F15.8)
IF(P(KH).LT.0.000001) GO TO 1101
4798 CONTINUE
1101 KS=KH
5400 AVGAVL = CI - AVGOUT - CCI
IF(JSW.EQ.1) GO TO 1007
C READ THE MONTHLY AVRAGE AND STANDARD DEVIATION
C EVRY MONTH HAS FIVE SLOPES
I=1
4014 CONTINUE
X(I)=N+I
JS = 1
1217 JB=4*JS-3
IZ =5*JB-2
COMPB= SLOPE(IZ)*(X(I)- SLOPE(IZ+1)) + SLOPE(IZ+2)
AVPEKB(JS) = EXP(COMPB)
W=AVPEKB(JS)
SIGMMN=W*SIGVRG(JS)
PEAK(JS)=AVPEKB(JS)+ 1.89*SIGMMN
1007 SIGMRG=SQRT(SIGMMN*SIGMMN+SIGMLS)
IF(JS.GT.1) GO TO 7444
CALL EXPPR(KS,CM,P,C,SLP,7)
7444 CONTINUE
WRITE(3,7896) C(7),SLP(7),SIGMRG
7896 FORMAT(' ',10X,E20.4,2F20.8)
WRITE(3,8794) SIGMMN,W,PEAK(JS)
8794 FORMAT('0',10X,3F20.8)
Program 5 (Cont.)

```

```

GG=C(7)
HH=SLP(7)
IF(JSW.EQ.1) GO TO 4479
AVPRM=FUNP(GG,HH,W)
DO 7150 IW=1,10
BA=COFSIG(IW)*SIGMMN
W1=W+BA
W2=W-BA
WRITE(3,7899) AVPRM,W1,W2
7899 FORMAT(' ',20X,'$$$$$',3F20.8)
AVPRM=AVPRM+FUNP(GG,HH,W1)+FUNP(GG,HH,W2)
7150 CONTINUE
RISK(JS)=AVPRM/21.0
WRITE(3,5571) AVPRM,RISK(JS)
5571 FORMAT('1',20X,' AVRG. PROB. = ',F20.9,' MONTHLY PROB. = ',F20.9)
RISKY=RISKY+RISK(JS)
IF(JS.NE.12) GO TO 5015
RISKYR=RISKY/12.0
WRITE(3,9547) RISKYR
9547 FORMAT(' ',20X,' YEARLY RISK = ',F15.9)
GO TO 5015
4479 AVGMRG = AVGAVL - AVPEKB(JS)+CAPURC
B = AVGMRG - 4.*SIGMRG
IF(B) 1010,1004,1004
1010 XL=AVGMRG/SIGMRG
XU=4.0
EPS=.00001
CZ=1.0/SQRT(2.0*3.1415927)
FCT=FUNC(Z)
CALL QATR(XL,XU,EPS,500,FCT,Y,IER,AUX)
Y=CZ*Y
RISK(JS)=Y
IF(JSW.EQ.1) GO TO 2050
RISKYR=RISKYR+Y
RIS=RISKYR
GO TO 1004

```

Program 5 (Cont.)

```

2050 RISKYR=RIS+Y
1004 IF(RISKYR.GT.RISKLV) GO TO 1008
      IF(JSW.EQ.1) GO TO 2070
      JS=JS+1
      IF(JS.NE.13) GO TO 5015
      JS = 1
      IRT = IRT + 1
      RISKOL =RISKYR
      I=I+1
      GO TO 4014
1008 IADD=JS
      IF(JSW.EQ.1) GO TO 2060
      SIZED=PEAK(JS)-CI
      OCAP = SIZED
      JSW=1
      IM=1
      NEWS=1
      DD 403 KQ=1,6
      IF(OCAP.LT.NEWCAP(KQ)) GO TO 1178
403  CONTINUE
1178 KF=KQ
      OCAP=NEWCAP(KF)
      GO TO 1001
2060 OCAP=NEWCAP(KQ-1)
      KF=KQ-1
      GO TO 1001
2070 JJS=JJS+1
      NEWUNT(JJS)=OCAP
      IIBTU=IHEATR(KF)
      KO=KO+1
      OCAPC(KO)=OCAP
      JP(KO)=KO
C      *****
C      COMPUTING THE CAPACITY FACTORS AND THE ACTUAL FUEL COST
C      *****
5015 CALL LFSORT(OCAPC, KK, ANFUEL)

```

Program 5 (Cont.)

```

      KKL=KKL+1
      J=KKL
      J3=4*(J-1)+1
      Y1=AY(J3)*PEAK(JS)
      Y2=AY(J3+1)*PEAK(JS)
      AA=0.
      ABB=0.
      BB=0.
      BSLOPE = Y2 - Y1
      M=KK
      I4=0
5002 IF(M.EQ.0) GO TO 4567
      ZZ=OCAPC(M)
      I4=I4+1
      IF(I4.LT.50) GO TO 5527
      WRITE(3,5573)
5573 FORMAT(1H1)
      I4=0
5527 CONTINUE
      IF(OCAPC(M).LT.200) GO TO 5610
      GO TO(1,2,3,5),I4
      1 FACTLD(M)=.50
      GO TO 4
      2 FACTLD(M)=.80
      GO TO 4
      3 FACTLD(M)=.85
      GO TO 4
      5 FACTLD(M)=.90
      4 AA=AA+ZZ*FACTLD(M)
      NEWUNT(1)=200
      JJS=1
      GO TO 5612
5610 AA=AA+ZZ*.9
      FACTLD(M) = 0.9
5612 CONTINUE
      IF(AA.GT.Y2) GO TO 5001

```

Program 5 (Cont.)

```

      BB = AA
      WRITE(3,5107) M,OCAPC(M),ANFUEL(M),FACTLD(M)
      M=M-1
      GO TO 5002
5001 IF(BB.GT.Y2) GO TO 5003
      BBB = Y2 - BB
      CCC = AA - Y2
      DDD(M) = (AA-Y1)/BSLOPE
      EEE(M) = BBB + (DDD(M) + 1.0)*CCC*0.5
      FACTLD(M) = EEE(M)/ZZ
      WRITE(3,5107) M,OCAPC(M),ANFUEL(M),FACTLD(M)
      BB = AA
      M=M-1
      GO TO 5002
5003 CCD = Y1 - AA
      DDD(M) = (AA-Y1)/BSLOPE
      EEE(M) = (DDD(M) + DDD(M+1))*ZZ*0.45
      FACTLD(M) = EEE(M)/ZZ
      WRITE(3,5107) M,OCAPC(M),ANFUEL(M),FACTLD(M)
      M=M-1
      CCX = ZZ*0.9
      IF(CCD.LT.CCX) GO TO 5005
      GO TO 5002
5005 EEE(M) = DDD(M+1)*CCD*.5
      FACTLD(M) = EEE(M)/ZZ
      WRITE(3,5107) M,OCAPC(M),ANFUEL(M),FACTLD(M)
5107 FORMAT(' ',10X,'FFFFFFFF',2I10,2F20.8,'FFFFFFFF')
4567 CONTINUE
      IPP=M+1
      DJ 6010 KM=IPP,KK
6010 SUMFUL=SUMFUL+ANFUEL(KM)*FACTLD(KM)
      WRITE(3,5027) SUMFUL
5027 FORMAT(' ',20X,'COST OF FUEL=',F20.8)
      JS=JS+1
      IF(JS.NE.13) GO TO 1217
C      COST OF OPERATION AND MAINTENANCE
Program 5 (Cont.)

```

```

AF=(1.0+FB)**(YR-YZ)
COSTMP=COSTMP*AF
AIRT=IRT
XY=AIRT-YZ
FL=(1.0+FB)**XY
TOTCST=TOTCST+SUMFUL+COSTMP
WRITE(3,7117) TOTCST
IF(NEWS.NE.1) GO TO 6001
JS=1
GO TO 1457
C *****
C COMPUTING THE COST OF THE NEW UNITS ADDED
C *****
6001 CONTINUE
VV=NEWUNT(JJS)
CCY1=C(1)*EXP(SLP(1)*VV)
CCY2=CCY1*VV
WRITE(3,7117) CCY2
CCY3=CCY2*FL
WRITE(3,7117) CCY3
CCY3=CCY3*FIXC
CCY3=2.*CCY3
WRITE(3,7117) CCY3
C COST OF OPERATION AND MAINTENANCE FOR NEW UNIT
TOTCST=TOTCST+CCY3
WRITE(3,7117) TOTCST
XFF=AIRT-PWY
1457 AINT=(1.+AINR)**XFF
WRITE(3,7117) AINT
PWCOST(IRT)=TOTCST/AINT
WRITE(3,7117) PWCOST(IRT)
7117 FORMAT(' ',20X,'TOTAL COST =',F19.8)
1185 STOP
END
SUBROUTINE EXPPR(N,AU,AW,C,SLP,M)
DIMENSION AU(1500),AW(1500),SLP(20),C(20)

```

Program 5 (Cont.)

```

AN=N
SUM2=0.0
SUM3=0.0
SUM4=0.0
SUM5=0.0
DO 100 I=1,N
100 AW(I)=ALOG(AW(I))
DO 200 I=1,N
SUM2=SUM2+AU(I)
SUM3=SUM3+AW(I)
SUM4=SUM4+AU(I)*AU(I)
200 SUM5=SUM5+AU(I)*AW(I)
XAV=SUM2/AN
YAV=SUM3/AN
A=SUM4-SUM2*SUM2/AN
B=SUM5-SUM2*SUM3/AN
SLP(M)=B/A
D=-SLP(M)*XAV+YAV
C(M)=EXP(D)
RETURN
END
SUBROUTINE TREXP(N,AV,AZ,C,SLP,M)
DIMENSION AV(100),AZ(100),SLP(20),C(20)
AN=N
SUM2=0.0
SUM3=0.0
SUM4=0.0
SUM5=0.0
DO 100 I=1,N
100 AZ(I)=ALOG(AZ(I))
DO 200 I=1,N
SUM2=SUM2+AV(I)
SUM3=SUM3+AZ(I)
SUM4=SUM4+AV(I)*AV(I)
200 SUM5=SUM5+AV(I)*AZ(I)
XAV=SUM2/AN
Program 5 (Cont.)

```



```

YAV=SUM3/AN
A=SUM4-SUM2*SUM2/AN
B=SUM5-SUM2*SUM3/AN
SLP(M)=B/A
D=-SLP(M)*XAV+YAV
C(M)=EXP(D)
RETURN
END
SUBROUTINE LFSORT(L,ISIZE,A)
IMPLICIT INTEGER*2(L)
DIMENSION L(100),A(100)
IF(ISIZE-1) 200,200,10
10 M=ISIZE-1
DO 160 J=1,M
IMIN=J
K=J+1
DO 40 I=K,ISIZE
IF(L(I)-L(IMIN)) 30,40,40
30 IMIN=I
40 CONTINUE
IF(IMIN-J) 160,160,45
45 LTEMP = L(IMIN)
ATEMP=A(IMIN)
80 IMIN1 = IMIN-1
L(IMIN) = L(IMIN1)
A(IMIN) = A(IMIN1)
IMIN = IMIN1
IF(J+1-IMIN) 80,80,120
120 L(J) =LTEMP
A(J) = ATEMP
160 CONTINUE
200 RETURN
END

```

Program 5 (Cont.)

XVII. APPENDIX F. EFFECT OF NEW UNIT SIZE ON RELIABILITY  
AND SPINNING RESERVE OF THE SYSTEM

In this appendix, the effects of the new unit size on the spinning reserve are discussed. Two methods were used to compute the reliability index for the Iowa Pool Power System.

The first method uses the cumulative probability of the available capacity. The results are shown below for January 1970.

Case	Installed Capacity MW	Average Peak MW	Monthly Peak MW	Risk Index	Excess Capacity Available MW	Days Outage each 10 years
1	2360	2011.0	2330	0.086855	30	218.872
2	2560	2011.0	2330	0.0013414	230	3.381
3	2960	2011.0	2330	0.00001978	630	0.0498
4	2960	2011.0	2330	0.00003834	630	0.0966

Case 1 is the original system without any ties.

Case 2 is adding a tie of 200 MW capacity.

Case 3 is adding 2 units each of 200 MW.

Case 4 is adding one unit of 400 MW capacity.

This method measures the risk index only and does not give the spinning reserve for the system. It is clear that 2 units each with 200 MW capacity provide the maximum reliability index to serve the load. From the second case, we see the effect of adding a tie to the system will greatly improve the reliability index.

Using the second method we can compute the reliability index and at the same time give the spinning reserve necessary to hold this index of reliability constant. This method is better than the other and is more accurate (see Appendix E).

In this method, we hold the index of reliability at constant value and find the corresponding megawatt outage at that level. This value will be the spinning reserve for the system which should be maintained if the system reliability index is to be realized. The results are shown in Fig. F.1 where the relation between the megawatt outages and the interval between two successive outages of that magnitude are given. At an interval of 2520 days (i.e., risk index is 0.0003968), the megawatt outage is obtained. Then the installed capacity  $I$ , the monthly average peak  $\bar{Y}$ , standard deviation for that month as a ratio to the monthly average peak  $\frac{\sigma}{\bar{Y}}$  and the capacity of units on maintenance are all known. Suppose we assume that all units are in service. If the installed capacity exceeds the monthly peak plus the megawatt outage at the previous risk index, then the system is reliable and no generating addition is required.

On the other hand, if the installed capacity is less than the monthly peak plus the megawatt outage, new generation is required.

In order to investigate the effect of the unit size on both the reliability and the cost of power system, we have to assume that the cumulative probability of megawatt outage is expressed mathematically by the following equation as in

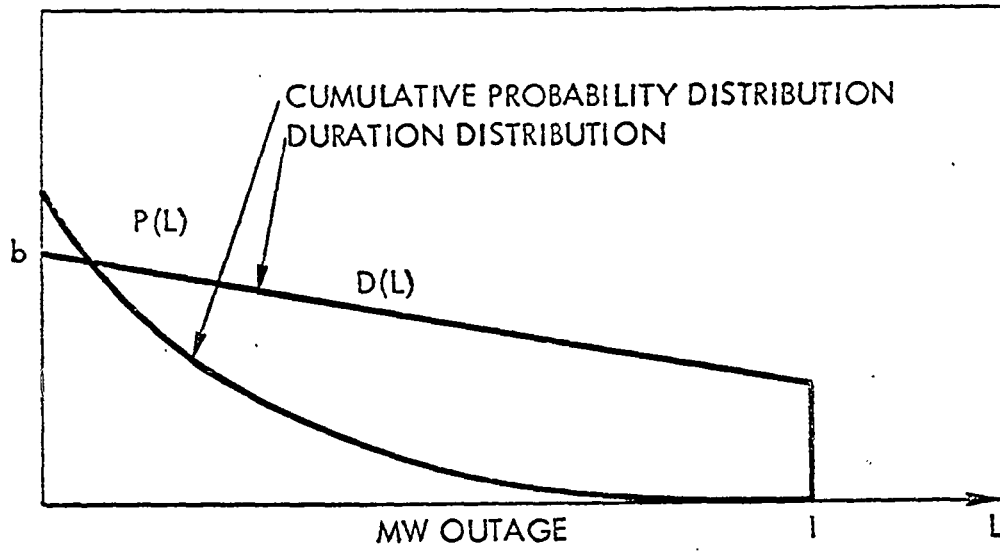


Fig. F.1. Cumulative probability and duration versus the megawatt outages

Appendix E,

$$P(L) = Ce^{mL} \quad (F.1)$$

Assume that the relation between the durations in days and the megawatt outages is approximated by a straight line relationship using the least square method. Let this relation be given as the following equation:

$$D(L) = aL + b \quad (F.2)$$

where  $D(L)$  is the duration in days of  $L$  megawatt outage,  $a$  is the slope of this straight line and  $b$  is duration of zero MW outage. Both functions are shown in Fig. F.1.

The probability density function of the megawatt outage can be obtained by differentiating equation F.1., i.e.,

$$p(L) = k_1 e^{mL} \quad (F.3)$$

where  $k_1 = Cm$ .

The frequency of an outage will be given as the ratio of the probability of this outage to the duration in days, i.e.,

$$F(L) = \frac{p(L)}{D(L)} \quad (F.4)$$

Assume we add a new unit of size  $C$  MW with a forced outage rate  $p$ , then the duration of this outage is  $D_c$  days and the interval between two successive outages is  $T_c$  days, on the original system defined by the above equations. This could be done in two steps as we have shown in Appendix E. The first step is to consider the unit in service with a probability  $q = p - 1$  and the probability of  $L$  megawatt outage, or

$$p_1(L) = qp(L) \quad (F.5)$$

where the index 1 refers to the first step. The duration of this outage will be given as

$$D_1(L) = \frac{D(L) \cdot qT_c}{D(L) + qT_c} \quad (F.6)$$

The second step is to consider the unit out of service with a probability  $p$  and the probability of  $L$  megawatt outage will be

$$p_2(L) = p \cdot p(L-C) \quad (F.7)$$

because we add the unit with its capacity  $C$  MW as an outage at this step.

Also, the duration of this outage is given as

$$D_2(L) = \frac{D(L-C) pT_c}{D(L-C) + pT_c} \quad (F.8)$$

Applying the addition law of probability, we can write the probability of an outage  $L$  as

$$p(L) = p_1(L) + p_2(L) \quad (F.9)$$

where the prime indicates the combined probability, or

$$p(L) = q \cdot p(L) + p \cdot p(L-C) \quad (F.10)$$

The frequency of such an outage will be given as

$$\hat{F}(L) = F_1(L) + F_2(L-C) \quad (F.11)$$

or

$$\hat{F}(L) = \frac{p_1(L)}{D_1(L)} + \frac{p_2(L)}{D_2(L)} \quad (F.12)$$

or

$$\hat{F}(L) = \frac{D(L) + qT_c}{D(L) \cdot T_c} p(L) + \frac{D(L-C) + pT_c}{D(L-C) T_c} p(L-C) \quad (F.13)$$

and the interval in days between two successive  $L$  MW outages will be given as

$$\hat{T}(L) = \frac{1}{\hat{F}(L)} \quad \text{days} \quad (F.14)$$

The relation between the intervals in days and  $L$ , the MW outage, is shown in Fig. F.2.

The first curve shows the interval for the original system while the second curve shows that for the system after adding a unit of size  $C$  MW.

For the risk index of 1 day per 10 years or  $R = 2520$  days as shown in Appendix E, we obtain the megawatt outage  $S$ . This value will be the spinning reserve for the system at this risk index. If the installed capacity is enough to supply the peak load demand and at the same time to provide the spinning reserve  $S$  MW, then the system is reliable and there is no need for new generation additions. On the other hand, if the system is not capable of doing so, we must add a new unit or units such that the risk index is not exceeded. Suppose we add a unit of capacity  $C$  MW to the system. At  $R = 2520$  days, the new spinning reserve value  $\hat{S}$  minus the old value  $S$  will give the excess in reserve ( $E$ ) due to the addition of this new unit, keeping the risk index at its constant value, i.e.,

$$E = \hat{S} - S \text{ MW} \quad (\text{F.15})$$

The new unit should supply this excess of reserve in case of emergency and the effective capability  $C_{ef}$  will be given as

$$C_{ef} = C - E \text{ MW} \quad (\text{F.16})$$

and the capacity factor  $F_c$  will be given as

$$F_c = \frac{\text{effective capability of the new unit (MW)}}{\text{the capacity of that unit (MW)}} \quad (\text{F.17})$$

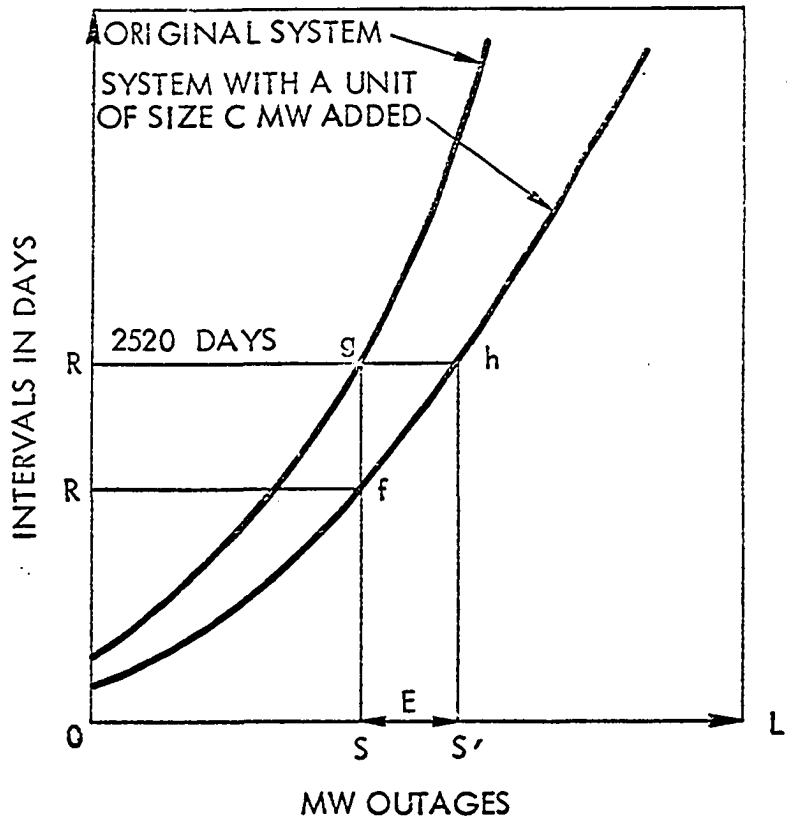


Fig. F.2. Intervals in days versus MW outages



or

$$F_C = \frac{C}{C} e^f \quad (\text{F.18})$$

In our analysis, our goal now is to choose a unit which will:

1. Improve the system reliability.
2. Minimize the increase of spinning reserve or maximize the effective capability of the new unit.
3. Optimize the cost of the addition generation.

If L is replaced by S (the spinning reserve) in equation F.13, we will have

$$\hat{F}(S) = \frac{D(S) + gT_C}{D(S) \cdot T_C} p(S) + \frac{D(S-C) + pT_C}{D(S-C) \cdot T_C} p(S-C) \quad (\text{F.19})$$

and equation F.14 can be rewritten as

$$\hat{T}(S) = \frac{1}{\hat{F}(S)} = \hat{R} \quad (\text{F.20})$$

Also, similar equations can be written for the original system at an outage equal to S MW, i.e.,

$$F(S) = \frac{p(S)}{D(S)} \quad (\text{F.21})$$

and

$$T(S) = \frac{1}{F(S)} = R \quad (\text{F.22})$$

Since the slope at point h is always less than that at point g, we can say that if the distance  $gf = R - \hat{R}$  days is minimized, the distance  $gh = S - \hat{S}$  MW will be also minimized. This minimization of the excess in spinning reserve will maximize the effective capability of the new unit added to the system

which will be given as

$$C_{ef} = C - gh \quad (F.23)$$

Since  $R$  is constant, we must maximize  $\hat{R}$  or minimize  $\hat{F}(S)$  given by equation F.19. This may be done by assuming any value for  $C$ , then computing  $S$  and repeating until  $\hat{F}(S)$  reaches its minimum value. This process is easy to apply but consumes much time. Since the parameters of the new units are not sensitive to size in the neighborhood of 200 MW, as shown in Table F.1., equation F.19 may be rewritten as follows:

$$\hat{F}(S) = k_0 + \frac{D(S-C) + pT_c}{D(S-C) \cdot T_c} p(S-C) \quad (F.24)$$

where  $k_0 = \frac{D(S) + qT_c}{D(S) \cdot T_c} p(S)$

This value is not going to change if the size variation is kept to a reasonable value. Using equations F.2 and F.3, equation F.24 can be rewritten as

$$\hat{F}(S) = k_0 + \left[ \frac{a(S-C)+b+pT_c}{a(S-C)+b)T_c} \right] k_1 e^{m(S-C)} \quad (F.25)$$

or

$$\hat{F}(S) = k_0 + \hat{k} \left[ \frac{d-aC}{1-aC} \right] e^{-mC} \quad (F.26)$$

where

$$\hat{k} = p(S)/T_c$$

$$d = aS+b+pT_c, \text{ and}$$

$$i = aS+b$$

Now, differentiating equation F.26 and equating to zero will give the following equation

Table F.1. Data for forced and scheduled outages

Fossil fuel units size	Forced outage rate %	Average duration of outages (days)	Average scheduled outage per year (weeks)
Less than 100MW	2	2	2
100MW to 250MW	2.5	2	2.5
250MW to 600MW			
Immature (first 3 years of operation)	4.5	4	3.0
Mature	3.0	3	3.0
600MW to 900MW			
Immature	5.75	4	4.0
Mature	3.75	3	4.0
Over 900MW			
Immature	6.75	5	4.0
Mature	4.50	4	4.0
<u>Nuclear units</u>			
800MW or less			
Immature	4.5	6	4.0
Mature	3.0	4	4.0
Above 800MW			
Immature	5.75	6	4.0
Mature	4.50	4	4.0

$$\frac{d\hat{F}(S)}{dC} = \hat{k} \left[ \frac{d-aC}{1-aC} \right] - m\hat{e}^{-mC} + \hat{k}\hat{e}^{-mC}$$

$$\left[ \frac{(1-aC)(-a) - (d-aC)(-a)}{(1-aC)^2} \right] = 0 \quad (\text{F.27})$$

Since  $\hat{e}^{-mC}$  is not equal to zero, then

$$\left[ \frac{d-aC}{1-aC} (-m) + \frac{a(d-1)}{(1-aC)^2} \right] = 0$$

If  $1 \neq ac$ , then we can write

$$m(1-aC)(d-aC) - a(d-1) = 0 \quad \text{or}$$

$$ma^2C^2 - ma(1+d)C - a(d-1) + mld = 0$$

or

$$\alpha C^2 + \beta C + \gamma = 0 \quad (\text{F.28})$$

where

$$\alpha = ma^2, \quad \beta = -ma(1+d) \quad \text{and}$$

$$\gamma = mld - a(d-1).$$

Equation F.28 is quadratic in C and has the solutions.

$$C_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad (\text{F.29})$$

The correct answer will be the larger value of C in equation F.29 which will minimize the distance  $gf = R - \hat{R}$  and at the same time keeps the increase in the spinning reserve to a minimum.

If we inspect equation F.19 closely, we can predict the effect of the size of new unit to the system. Since the first term on the right hand side of this equation is not sensitive

to size variation, we will concentrate on the second term which could be rewritten as

$$F(S-C) = \frac{D(S-C)+pT_c}{D(S-C) \cdot T_c} p(S-C) \quad (F.30)$$

Assume that we select the size  $C$  of the new unit to be exactly equal to  $S$ . Equation F.30 will become

$$F(o) = \frac{D(o)+pT_c}{D(o) \cdot T_c} F(o) \quad (F.31)$$

But the probability of zero outage is the largest value and is equal to  $q^n$  for a system of  $n$  units having a probability of being in service equal to  $q$  (close to unity) and  $F(o)$  will be large enough to increase the required spinning reserve. As  $C$  decreases than  $S$ , the probability term decreases more rapidly than the increase in the duration term in equation F.31 resulting in a substantial reduction in the frequency.

On the other hand, if  $C$  increases than  $S$ , equation F.19 may be rewritten as

$$\hat{F}(S) = \frac{D(S)+qT_c}{D(S) \cdot T_c} p(s) \quad (F.32)$$

provided that the second term is zero and the value of  $S-C$  becomes negative. Then equation F.32 may be rewritten as

$$\hat{F}(S) = \frac{D(S)+qT_c}{T_c} F(S) \quad (F.33)$$

or  $\hat{F}(S)$  will be less than  $F(S)$  since  $q$  is close to unity while  $D(S)$  is a small value compared to  $T_c$ . This gives the impression of improving the reliability index which is not true. This

could be explained if we compute the frequency for an outage equal to the unit size C.

$$\hat{F}(C) = \frac{D(C)+qT_C}{D(C)T_C} p(C) + \frac{D(o)+pT_C}{D(o) \cdot T_C} p(o)$$

or

$$\hat{F}(C) = \frac{D(C)+qT_C}{T_C} F(C) + \frac{D(o)+pT_C}{T_C} F(o) \quad (F.34)$$

Since  $F(o)$  and  $D(o)$  are the highest values of the frequencies and the durations, it is more likely that  $\hat{F}(C)$  given by equation F.34 will be higher than  $F(S)$  resulting in an increase in the spinning reserve, i.e., the interval between two successive outages equal to the capacity C MW will be less than R days as shown in Fig. F.2.

The conclusion is that the size of the new unit added to the system has great impact on the reliability index, the spinning reserve, and the economy of the system. A large unit added to the system appears to be attractive from the economical point of view. However, a large unit decreases the reliability index and also increases the spinning reserve considerably.

In Fig. F.3, four cases are shown as follows:

Case 1 is the original Pool system with 2360 MW capacity.

Case 2 is adding a tie of 200 MW capacity.

Case 3 is adding two units of 200 MW capacity each.

Case 4 is adding a 400 MW unit.

Since we consider the tie has a 100% reliability compared to generators, the first two cases will have the same identical

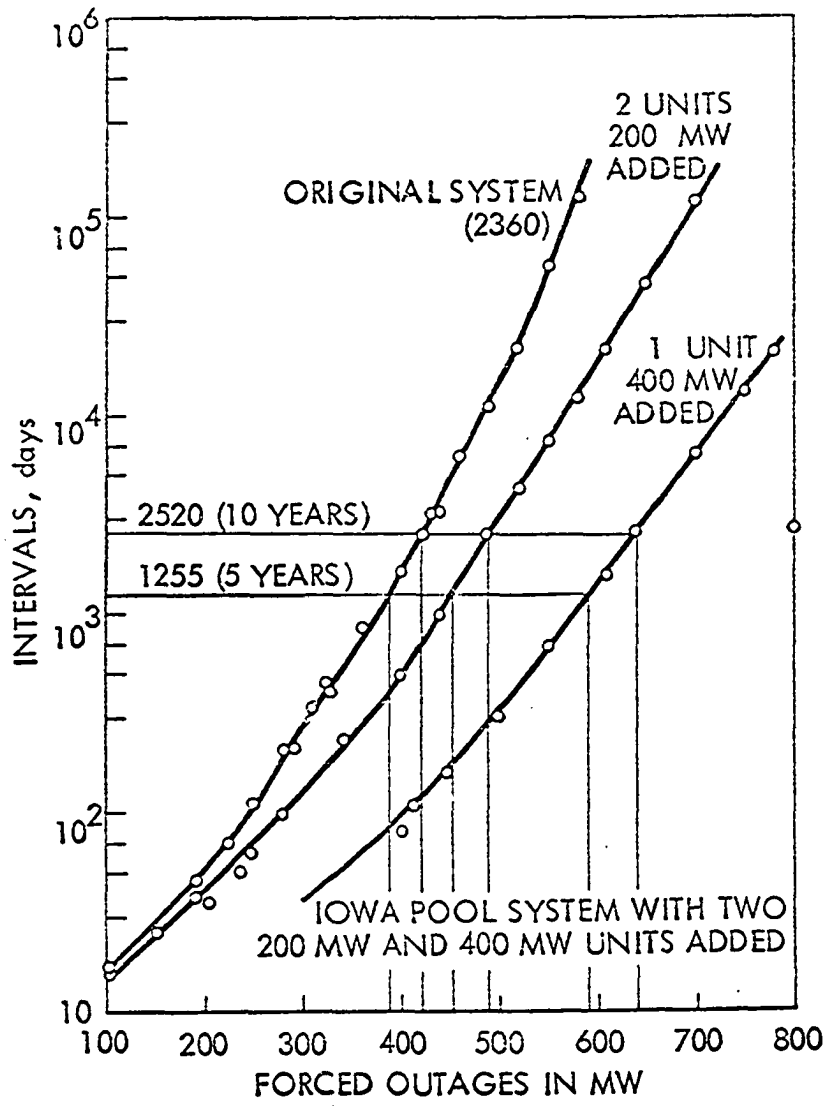


Fig. F.3. Iowa Pool system with two 200 MW and 400 MW units added

curve and the effect of the tie will only result in an increase in the installed capacity without changing the reserve required by the system to have the same predetermined reliability index. That explains the reason for having three curves in Fig. F.3.

A summary of the results are shown, first for a reliability index of 1 day per 10 years, in Table F.1., then

Table F.2. Summary of four cases with reliability index of 1 day per 10 years

Case	Installed capacity MW	Monthly peak MW	Spinning reserve MW	Demand MW	Net capability
1	2360	2330	420	2750	-390
2	2560	2330	420	2750	-190
3	2960	2330	490	2820	+140
4	2960	2330	640	2970	-10

for a reliability index of 1 day per 5 years in Table F.3.



Table F.3. Summary of four cases with reliability index of 1 day per 5 years

Case	Installed capacity MW	Monthly peak MW	Spinning reserve MW	Demand MW	Net capability
1	2360	2330	390	2720	-360
2	2560	2330	390	2720	-160
3	2960	2330	453	2783	+177
4	2960	2330	590	2920	+40

To find the risk index, we must subtract the monthly peak load from the installed capacity and from Fig. F.3 or Table E.17 of Appendix E, we could find the risk index as shown in Table F.4.

Table F.4. Risk indices for four cases

Case	Installed capacity MW	Monthly peak MW	MW outage available	Interval in days	Risk index
1	2360	2330	30	12	0.0833000
2	2560	2330	230	80	0.0125000
3	2960	2330	630	34,000	0.0000265
4	2960	2330	630	2,520	0.0003968

It is clear that the 2 units each of 200 MW are superior than the one unit of 400 MW from the reliability point of view.

Now we would like to compute the effective capability of the new added unit. For example, consider the 400 MW unit. The system capacity before adding this unit is 2560 MW with a spinning reserve equal to 420 MW for a risk index of 1 day per 10 years.

After adding the unit the reserve is 640 MW, Excess of reserve =  $640 - 420 = 220$  MW. Then the new unit should carry a load equal to its capacity minus the new excess of spinning reserve due to this addition, i.e., effective load carried by the new unit will be  $400 - 220 = 180$  MW, or the effective load capability will be 180 MW. The capacity factor is

$$F_c = 100 \times \frac{180}{400} = 45\%$$

This is very poor and means that a unit of 400 MW will have an effective load capability of only 180 MW which is not economically acceptable. Consider the 2 units each of 200 MW. Then we compute

$$\text{Excess of reserve} = 490 - 420 = 70 \text{ MW}$$

$$\text{Effective load capability of both units} = 400 - 70 = 330$$

$$\text{The capacity factor for both units} = 100 \times \frac{330}{400} = 82.5\%$$

Now the same could be done for 1 day for 5 years as a risk index. Table F.5 shows the results of both risk indices.

Table F.5. Comparison of capacity factors for different risk indices

Capacity MW	Excess reserve MW	Effective load capability MW	Capacity factor %	Risk index
400	220	180	45.0	1 day/10 years
2 x 200	70	330	82.5	"
400	195	205	51.25	1 day/ 5 years
2 x 200	60	340	85.0	"

To investigate the effect of the new unit size on both the reliability and the spinning reserve of the Iowa Pool system, three expansion patterns were studied. In the first expansion pattern, three units of 200 MW each were added. In the second pattern, three units of 300 MW each were added, while in the third three 400 MW units were added.

For a risk index of 1 day per 10 years, the effective capability of each unit in each pattern was computed. The capacity factors as well as the spinning reserve of the system were also computed and the results are plotted in Figures F.4 to F.8.

For a risk index of 1 day per 5 years, the same calculations were performed and the results are plotted in Figures F.9 and F.10.

In Fig. F.4, the interval in days between two successive outages is plotted against the MW outage. Four curves are

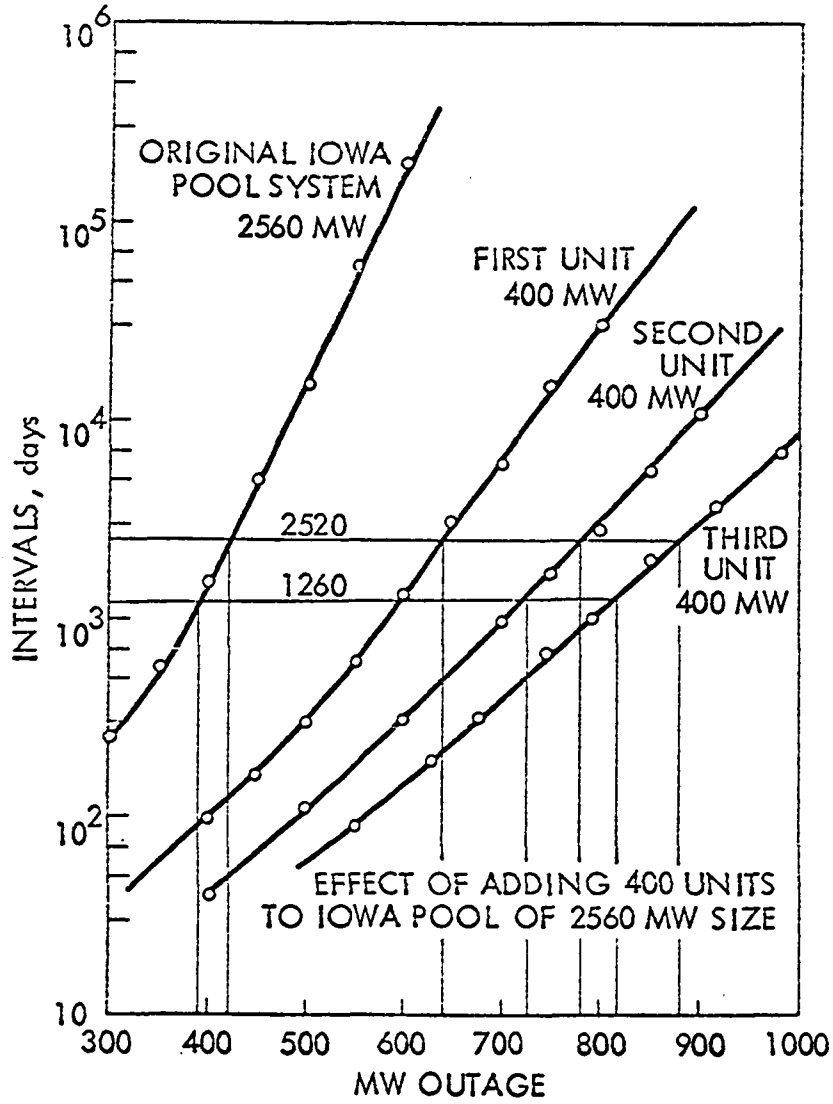


Fig. F.4. Effect of adding 400 units to Iowa Pool of 2560 MW size

shown. The first curve shows the original Iowa Pool system. The second curve shows the effect of adding one unit of 400 MW capacity. The third curve shows the effect of adding a second unit of the same size while the fourth curve shows the effect of the third 400 MW unit addition. In Fig. F.5, the capacity factors for the three patterns against the number of units added to the system. Fig. F.6 shows the spinning reserve as a percentage of the installed capacity versus the number of units. Fig. F.7 shows the capacity factors versus the unit size in MW. The other figures are similar to the previous ones except that the risk index is 1 day per 5 years.

From the above results, the 200 MW unit pattern is better than the other two expansion patterns because the units have the highest capacity factors and at the same time the least spinning reserve. The influence of the new unit size addition on the reliability and the spinning reserve depends primarily on the size of the original system and average size of the units in that system. For Iowa Pool system, the average unit size is approximately 35 MW which is small compared to 400 MW unit. Also, the 400 MW with its higher forced outage rate required a high percent of spinning reserve which results in a low effective capability.

For a system like the Iowa Pool, the 200 MW unit is more convenient than the 400 MW unit if we are considering the Pool by itself. However, the 400 MW unit may be more suitable if we consider the other ties connecting the Iowa Pool to the outside system.

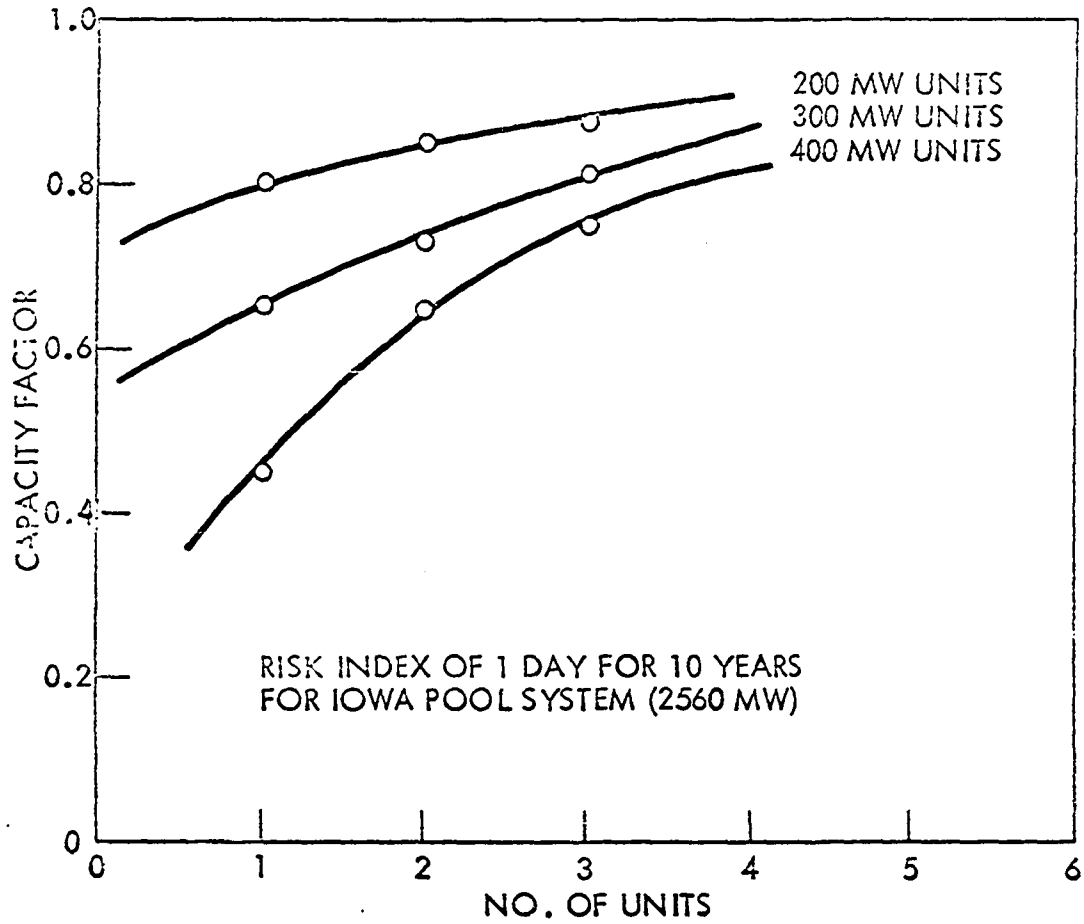


Fig. F.5. Risk index of 1 day for 10 years for Iowa Pool system (2560 MW)

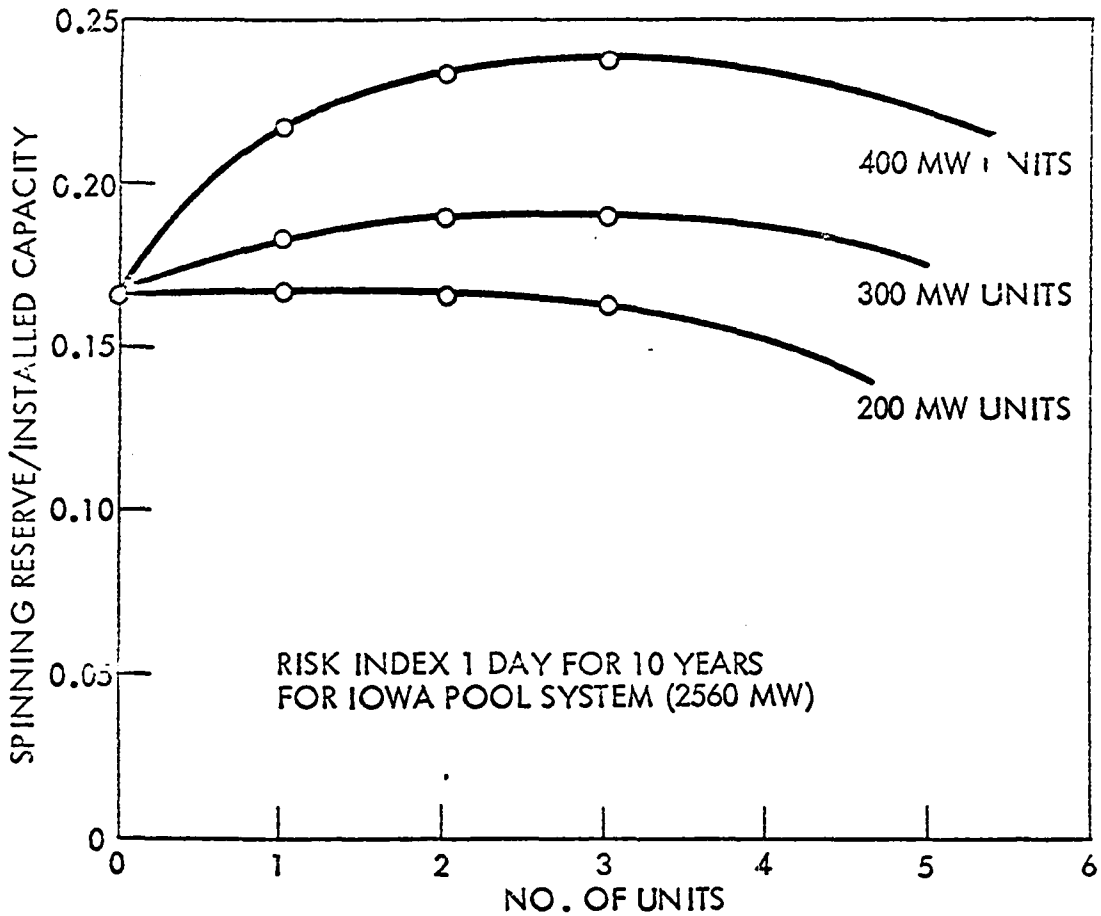


Fig. F.6. Risk index 1 day for 10 years for Iowa Pool system (2560 MW)

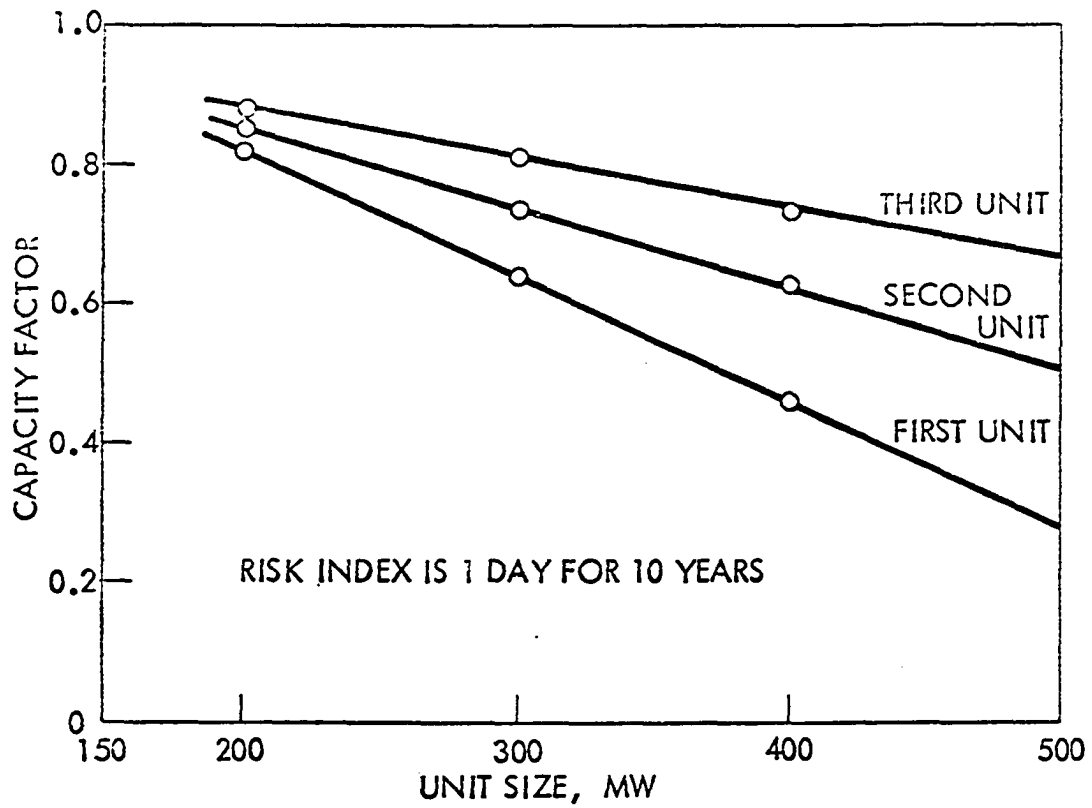


Fig. F.7. Risk index is 1 day for 10 years



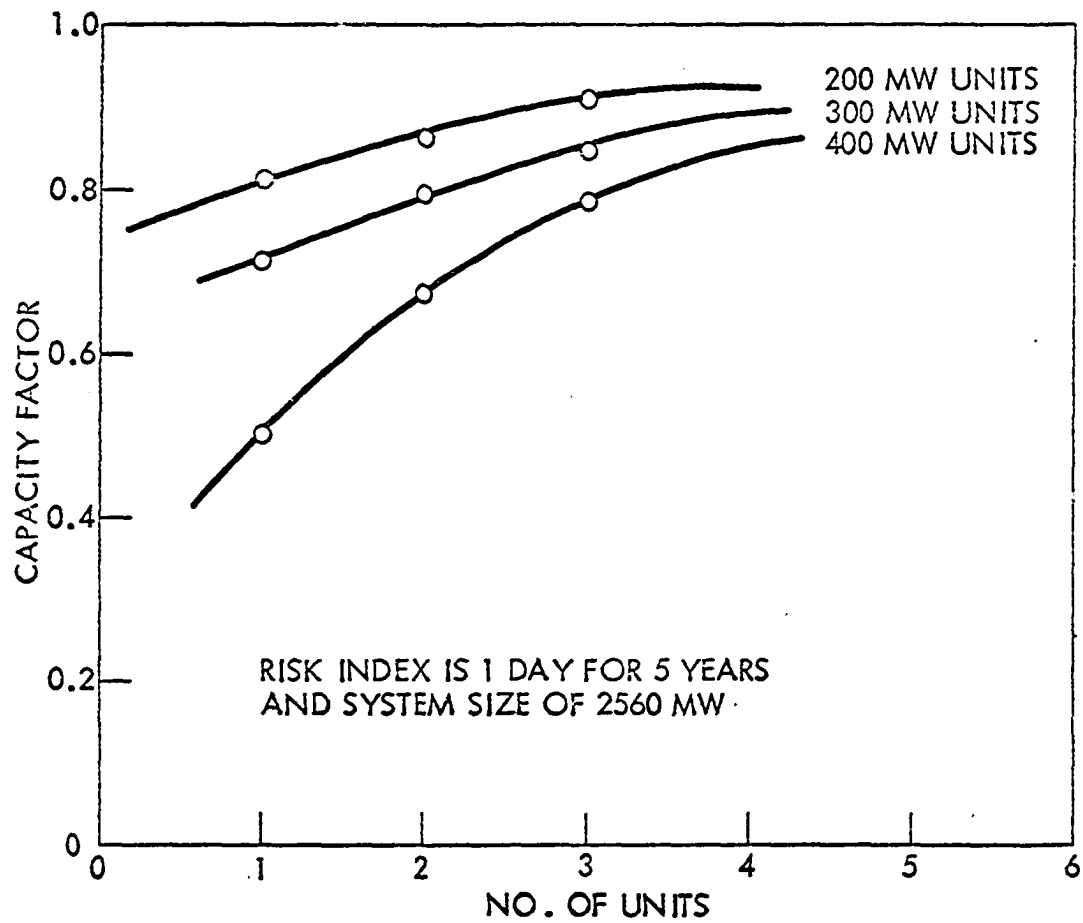


Fig. F.8. Risk index is 1 day for 5 years and system size of 2560 MW

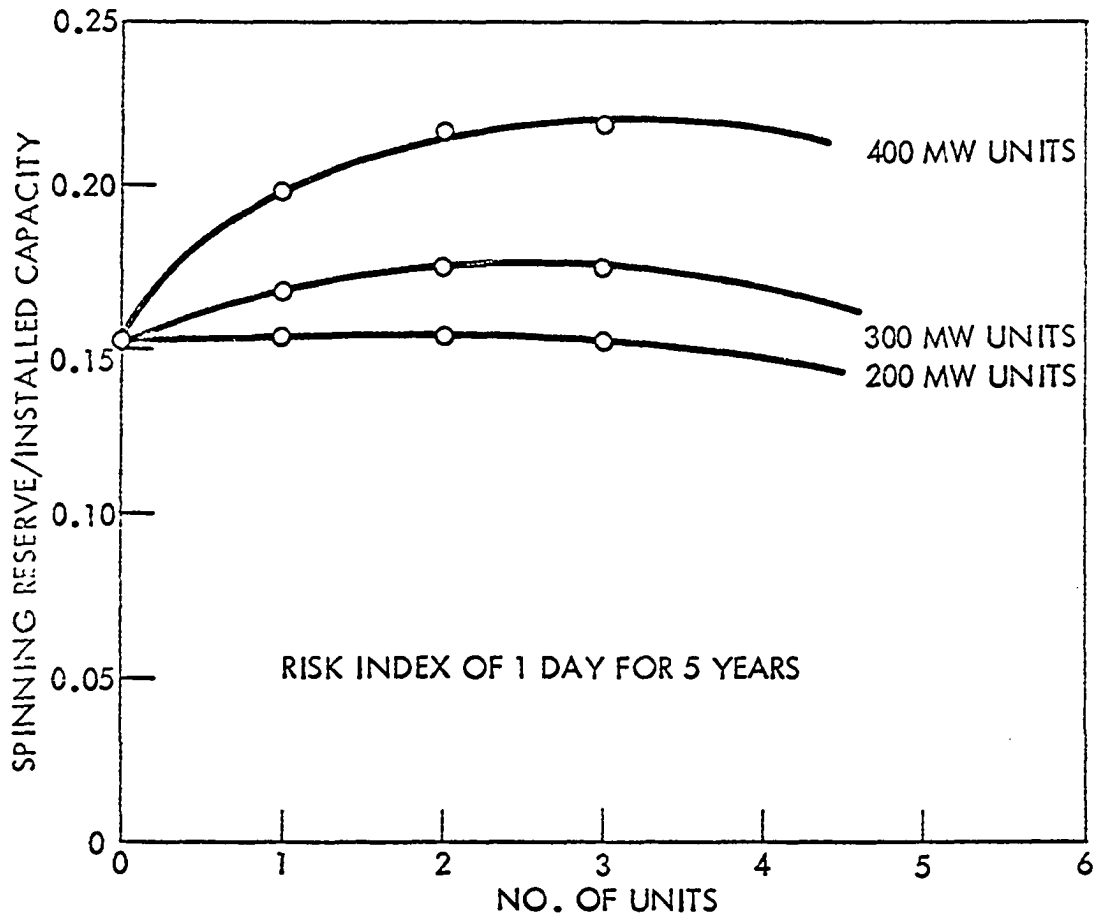


Fig. F.9. Risk index of 1 day for 5 years

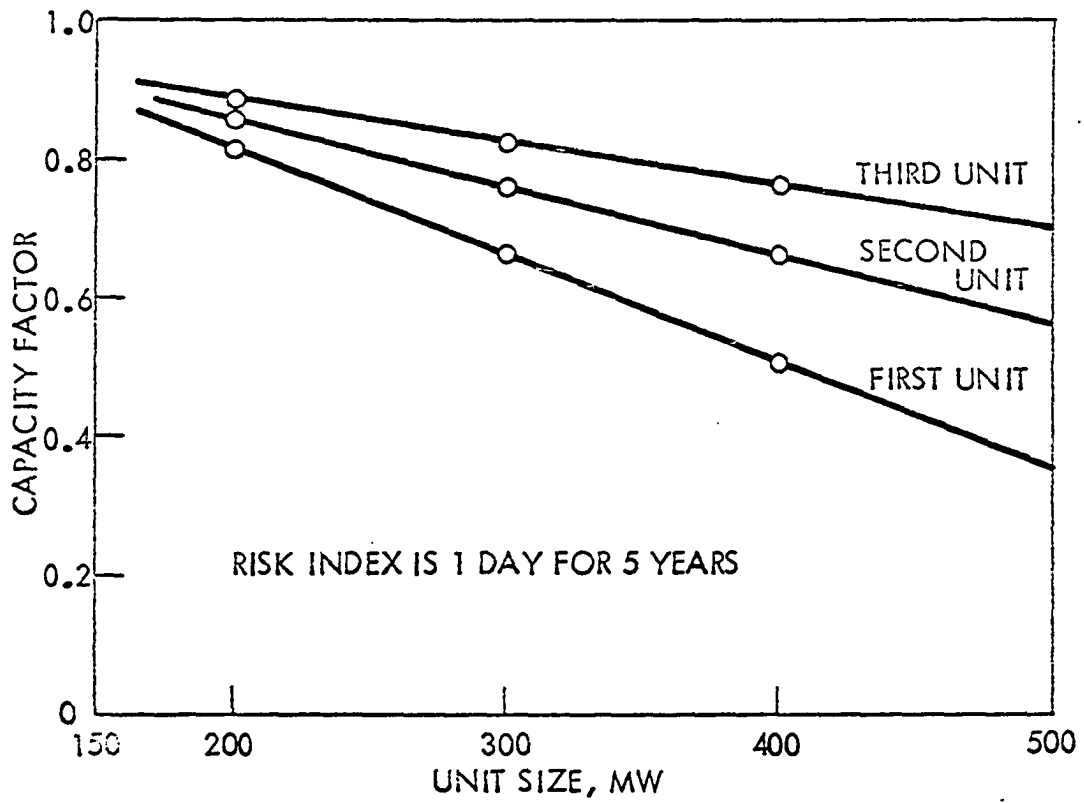


Fig. F.10. Risk index is 1 day for 5 years

## XVIII. APPENDIX G. COST PROGRAM

In this appendix, the capacity data for the Iowa Pool is shown in Table G.1. In the first column, a serial number is shown which will identify the company that owns this unit. This serial number is coded according to the following code:

<u>First digit of serial number</u>	<u>Company name</u>	<u>Code name</u>
1	Iowa Public Service	IPS
2	Iowa Electric Light and Power	IELP
3	Iowa Power and Light	IPL
4	Iowa Southern Utility	ISU
5	Iowa-Illinois Gas and Electricity	IIGE
6	Cornbelt Power Cooperative	CRNB

The name for that unit appears in the second column. The heat rate (Btu/Kw-hr) and the fuel cost (cent/10<sup>6</sup>Btu) are shown on columns three and four respectively. The fuel cost (k\$) for the units on full load base are shown in the last column.

Table G.2 shows the annual operating and maintenance costs (k\$) for the same units in Table G.1.

The capacity factors of these units are shown in Table G.3.

The investment costs in \$/Kw for one unit is plotted in Fig. G.1 while Fig. G.2 shows the same costs if two units of the same size will be added at the same locations. The annual operation and maintenance costs for one unit and two units of the same size are shown in Fig. G.3 and Fig. G.4 respectively.

Table G.1. Fuel cost for Iowa Pool units on full load basis

Serial No.	Name	Capacity MW	Heat rate Btu/Kw-hr	Fuel cost cent/10 <sup>6</sup> Btu	Cost in thousands of dollars
101	Neal 1	150	9,800	23.0	239.257400
102	Maynard 7	60	10,800	28.0	98.725200
103	Maynard 6	25	10,800	28.0	55.188000
104	Kirk 5	10	12,800	26.5	26.163190
105	Kirk 1	10	17,200	26.5	33.273390
106	Eagle Grove 1	10	13,800	26.0	26.192390
107	Hawkeye 2	15	13,800	30.1	45.484100
108	Charles City	30	15,000	39.0	128.115000
109	Hawkeye 1	10	15,000	30.0	32.959480
110	Carroll 1	5	16,250	29.4	17.437850
111	Carroll 2	5	18,000	29.4	19.315790
201	P. Creek 4	140	9,700	29.6	293.436200
202	Sutherland 3	80	10,150	27.9	165.380000
203	P. Creek 3	50	11,100	29.6	119.924400
204	Sutherland 2	35	12,450	27.9	88.749200
205	Sutherland 1	35	12,450	27.9	88.749200
206	P. Creek 2	25	12,500	29.3	66.840620
207	P. Creek 1	25	12,500	29.3	66.840620
208	Six. St. 1	35	14,200	29.6	107.391700
209	Six. St. 7	20	14,750	29.6	63.743600
210	Six. St. 4	15	14,750	29.6	47.807690
211	Boone 2	20	13,900	27.4	55.605560
212	Boone 1	10	15,250	27.4	30.503030
213	Iowa Falls 4	10	14,800	27.8	30.035110
214	Six. St. 6	10	17,200	29.6	37.165750
215	Six. St. 1 & 2	10	17,200	29.6	37.165750
216	Disel 8	10	12,000	28.5	24.965980
302	DPS No. 2-7	120	10,200	25.9	231.421400

Table G.1 (Cont.)

Serial No.	Name	Capacity MW	Heat rate Btu/kw-hr	Fuel cost cent/10 <sup>6</sup> Btu	Cost in thousands of dollars
303	Council B. 2	90	10,200	27.5	184.288400
304	Council B. 1	50	11,300	27.5	113.423700
305	DPS No. 2-6	70	11,300	25.9	149.554300
306	DPS No. 2-4	40	12,400	25.9	93.778710
307	DPS No. 2-5	50	12,600	25.9	119.114100
308	DPS No. 2-2	30	16,000	25.9	90.753600
309	River Hill 1	20	16,000	25.4	59.334390
310	River Hill 2	20	16,000	25.4	59.334390
311	River Hill 3	20	16,000	25.4	59.334390
312	River Hill 4	20	16,000	25.4	59.334390
313	River Hill 5	20	16,000	25.4	59.334390
314	River Hill 6	20	16,000	25.4	59.334390
315	River Hill 7	20	16,000	25.4	59.334390
316	River Hill 8	20	16,000	25.4	59.334390
317	DPS No. 2-1	15	17,300	25.9	49.063650
318	DPS No. 2-3	15	14,750	29.6	47.807690
401	Burlington 1	200	10,000	23.5	343.099600
402	Bridgeport 1	20	13,500	23.4	46.121390
403	Bridgeport 2	20	13,500	23.4	46.121390
404	Bridgeport 3	20	13,500	23.4	46.121390
502	Riverside 5	140	9,900	26.8	271.156700
503	Moline 7	25	13,000	28.2	66.904490
504	Riverside 3	5	15,000	26.8	14.672990
505	Riverside 4	50	11,800	26.8	115.427600
506	Moline 6	25	13,200	28.2	67.933800
507	Riverside 3	25	13,400	26.8	65.539390
508	Moline 5	20	13,400	28.2	55.170480
509	Coralvill 1	20	16,000	30.0	70.080000

Table G.1 (Cont.)

Serial No.	Name	Capacity MW	Heat rate Btu/kw-hr	Fuel cost cent/10 <sup>6</sup> Btu	Cost in thousands of dollars
510	Coralvill 2	20	16,000	30.0	70.080000
511	Coralvill 3	20	16,000	30.0	70.080000
512	Coralvill 4	20	16,000	30.0	70.080000
513	Moline G.T. 1	20	16,000	30.0	70.080000
514	Moline G.T. 2	20	16,000	30.0	70.080000
515	Moline G.T. 3	20	16,000	30.0	70.080000
516	Moline G.T. 4	20	16,000	30.0	70.080000
517	Riverside 6	20	16,000	27.5	64.240000
518	Riverside 7	20	16,000	27.5	64.240000
519	Riverside 8	20	16,000	27.5	64.240000
601	Wisdom 1	40	12,500	31.6	115.340000
602	Humboldt 4	20	14,150	35.4	73.132850
603	Humboldt 3	10	15,400	35.4	39.796670
604	Humboldt 2	10	18,000	33.8	44.413190
605	Humboldt 1	10	18,700	33.8	46.140360

The adopted coding makes it easy to track every unit and the individual company sharing in monthly fuel cost is computed easily as follows:

The monthly fuel cost for	IPS	=	722.1117	k\$
"	"	"	"	"
"	"	"	"	"
"	"	"	"	"
"	"	"	"	"
"	"	"	"	"
"	"	"	"	"
"	"	"	"	"

In this computation we assume that all units in the Pool are at full load.



Table G.2. Operation and maintenance costs

Serial No.	Capacity MW	Operation and maintenance cost in 10 <sup>3</sup> \$
101	150	654.3908
102	60	286.3073
103	25	123.5274
104	10	50.1548
105	10	50.1548
106	10	50.1548
107	15	74.8584
108	30	147.4963
109	10	50.1548
110	5	25.2020
111	5	25.2020
201	140	616.8791
202	80	374.2131
203	50	240.9782
204	35	171.2241
205	35	171.2241
206	25	123.5274
207	25	123.5274
208	35	171.2241
209	20	99.3153

Table G.2 (Cont.)

Serial No.	Capacity MW	Operation and maintenance cost in 10 <sup>3</sup> \$
210	15	74.85841
211	20	99.3153
212	10	50.1548
213	10	50.1548
214	10	50.1548
215	10	50.1548
216	10	50.1548
302	120	539.3935
303	90	416.8171
304	50	240.9782
305	70	330.7145
306	40	194.7125
307	50	240.9782
308	30	147.4963
309	20	99.3153
310	20	99.3153
311	20	99.3153
312	20	99.3153
313	20	99.3153
314	20	99.3153

Table G.2 (Cont.)

Serial No.	Capacity MW	Operation and maintenance cost in 10 <sup>3</sup> \$
315	20	99.3153
316	20	99.3153
317	15	74.8584
318	15	74.8584
401	200	830.1279
402	20	99.3153
403	20	99.3153
404	20	99.3153
502	140	616.8791
503	25	123.5274
504	5	25.2020
505	50	240.9782
506	25	123.5274
507	25	123.5274
508	20	99.3153
509	20	99.3153
510	20	99.3153
511	20	99.3153
512	20	99.3153
513	20	99.3153

Table G.2 (Cont.)

Serial No.	Capacity MW	Operation and maintenance cost in 10 <sup>3</sup> \$
514	20	99.3153
515	20	99.3153
516	20	99.3153
517	20	99.3153
518	20	99.3153
519	20	99.3153
601	40	194.7125
602	20	99.3153
603	10	50.1548
604	10	50.1548
605	10	50.1548
Total cost (k\$)		11,127.9400

Table G.3. Capacity factors  
for all units

Serial No.	Capacity MW	Capacity factor
401	200	0.9000
101	150	0.9000
201	140	0.9000
502	140	0.9000
302	120	0.9000
303	90	0.9000
202	80	0.8848
305	70	0.8391
102	60	0.7985
203	50	0.7641
307	50	0.7328
304	50	0.7016
505	50	0.6703
306	40	0.6422
601	40	0.6172
204	35	0.5937
205	35	0.5718
208	35	0.5500
308	30	0.5296
108	30	0.5109
206	25	0.4937
207	25	0.4781
506	25	0.4624
507	25	0.4468
206	25	0.4312
207	25	0.4155
209	20	0.4155
211	20	0.4015

Table G.3 (Cont.)

Serial No.	Capacity MW	Capacity factor
309	20	0.3765
310	20	0.3640
508	20	0.3515
509	20	0.3390
510	20	0.3265
602	20	0.3140
403	20	0.3015
402	20	0.2889
511	20	0.2764
512	20	0.2639
513	20	0.2514
514	20	0.2389
316	20	0.2264
318	20	0.2139
517	20	0.2010
518	20	0.1889

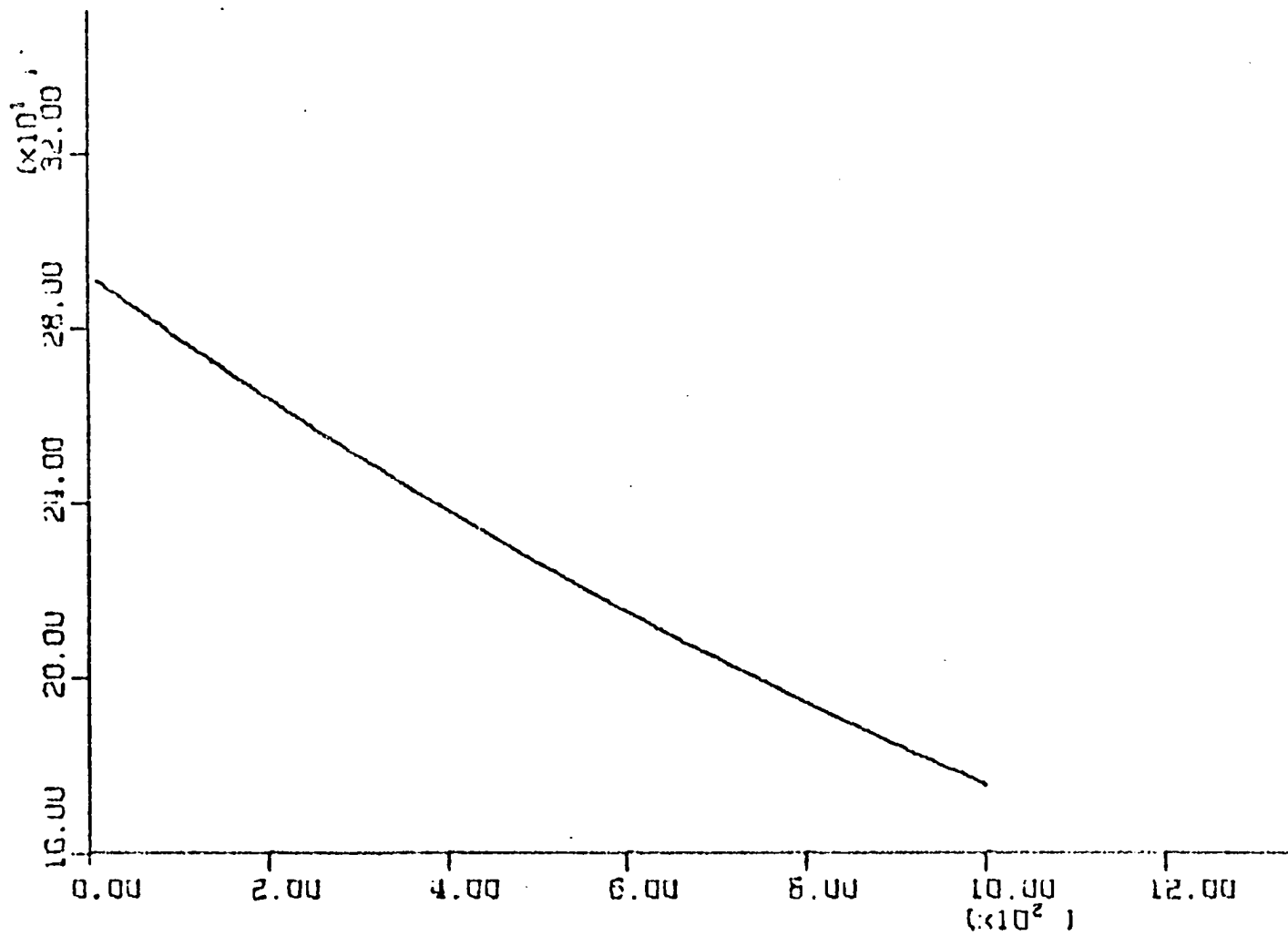


Fig. G.1. Generation investment cost for one unit

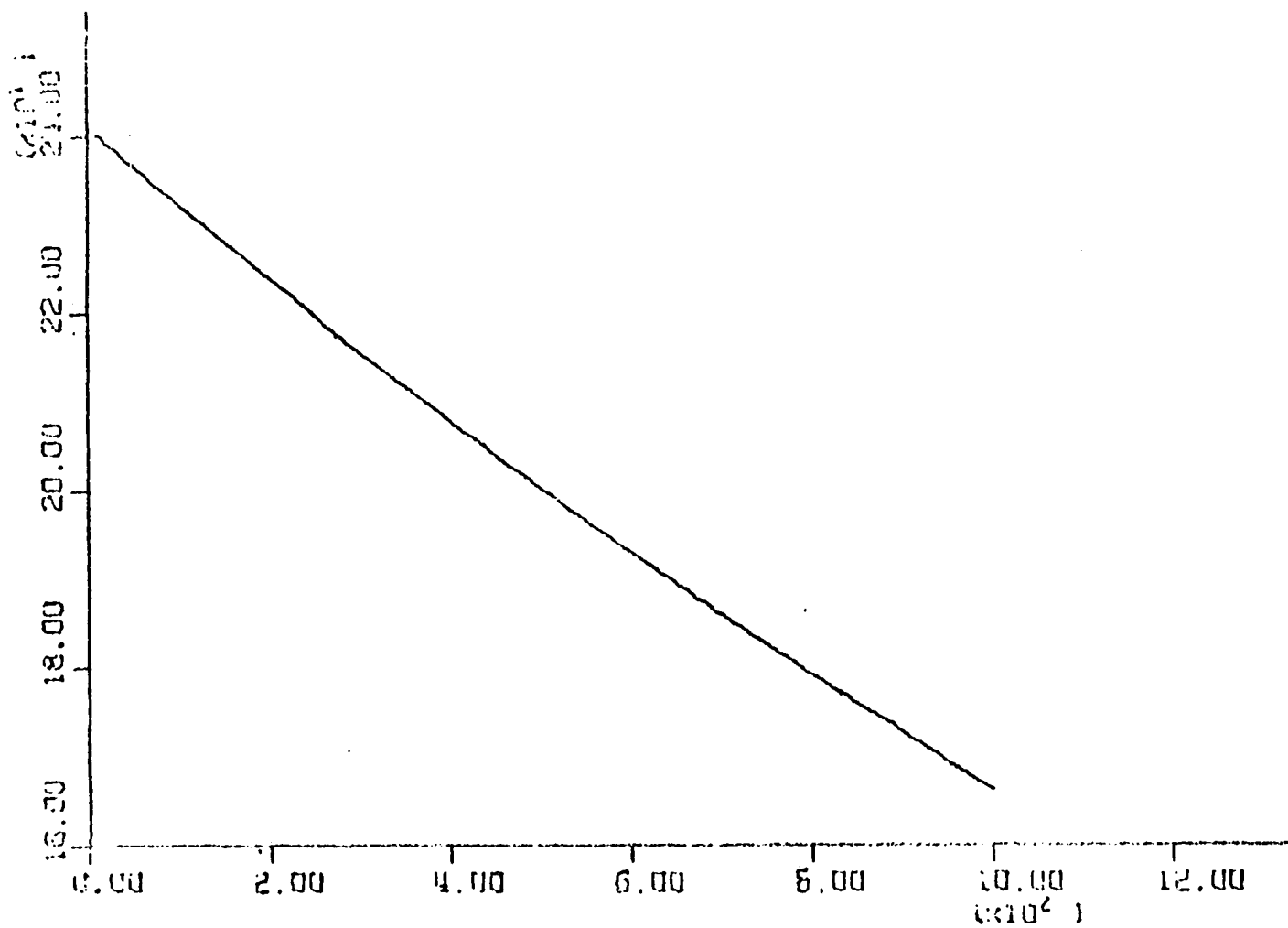


Fig. G.2. Generation investment cost for two units of the same size

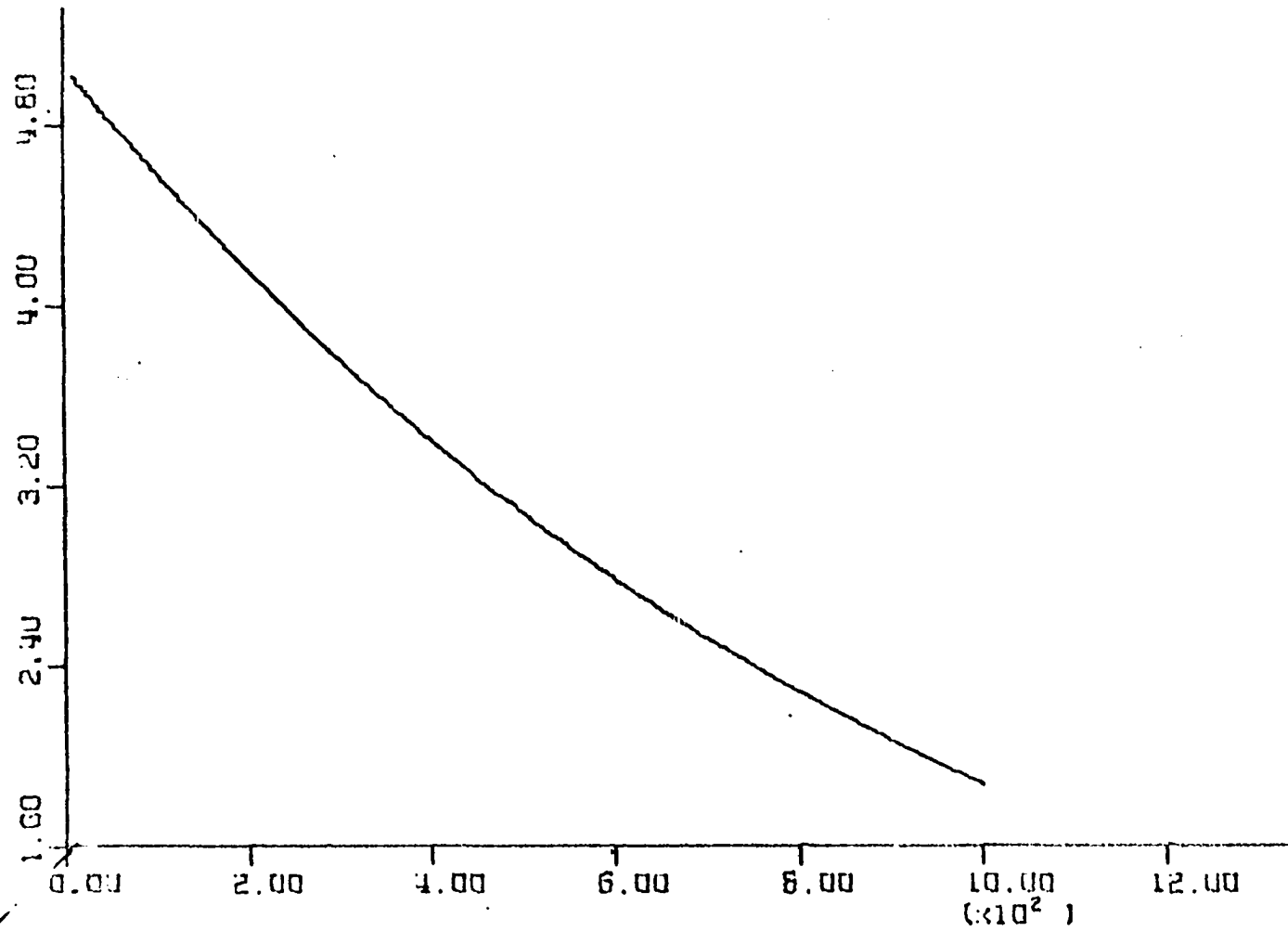


Fig. G.3. Operation and maintenance costs for one unit

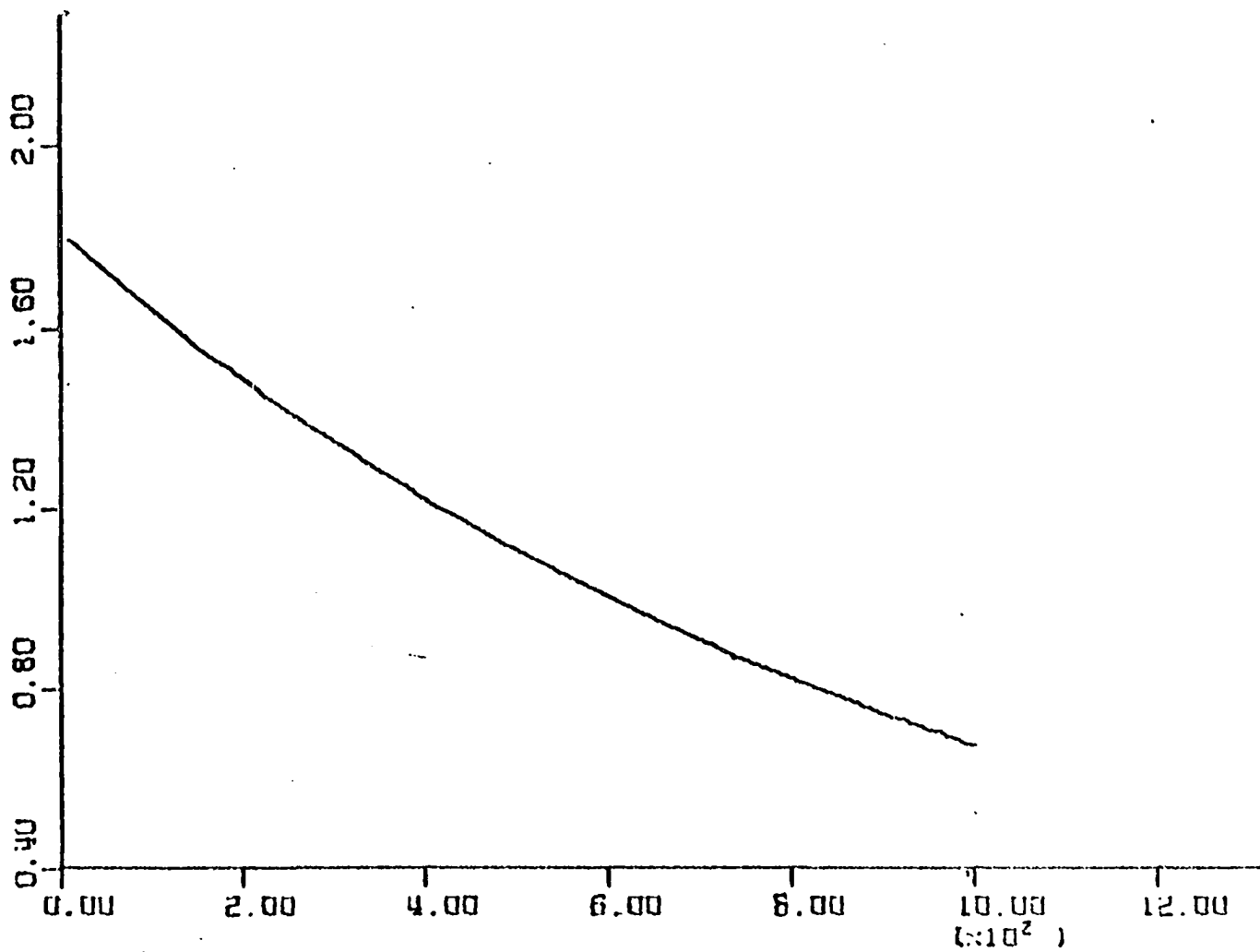


Fig. G.4. Operation and maintenance costs for two units of the same size

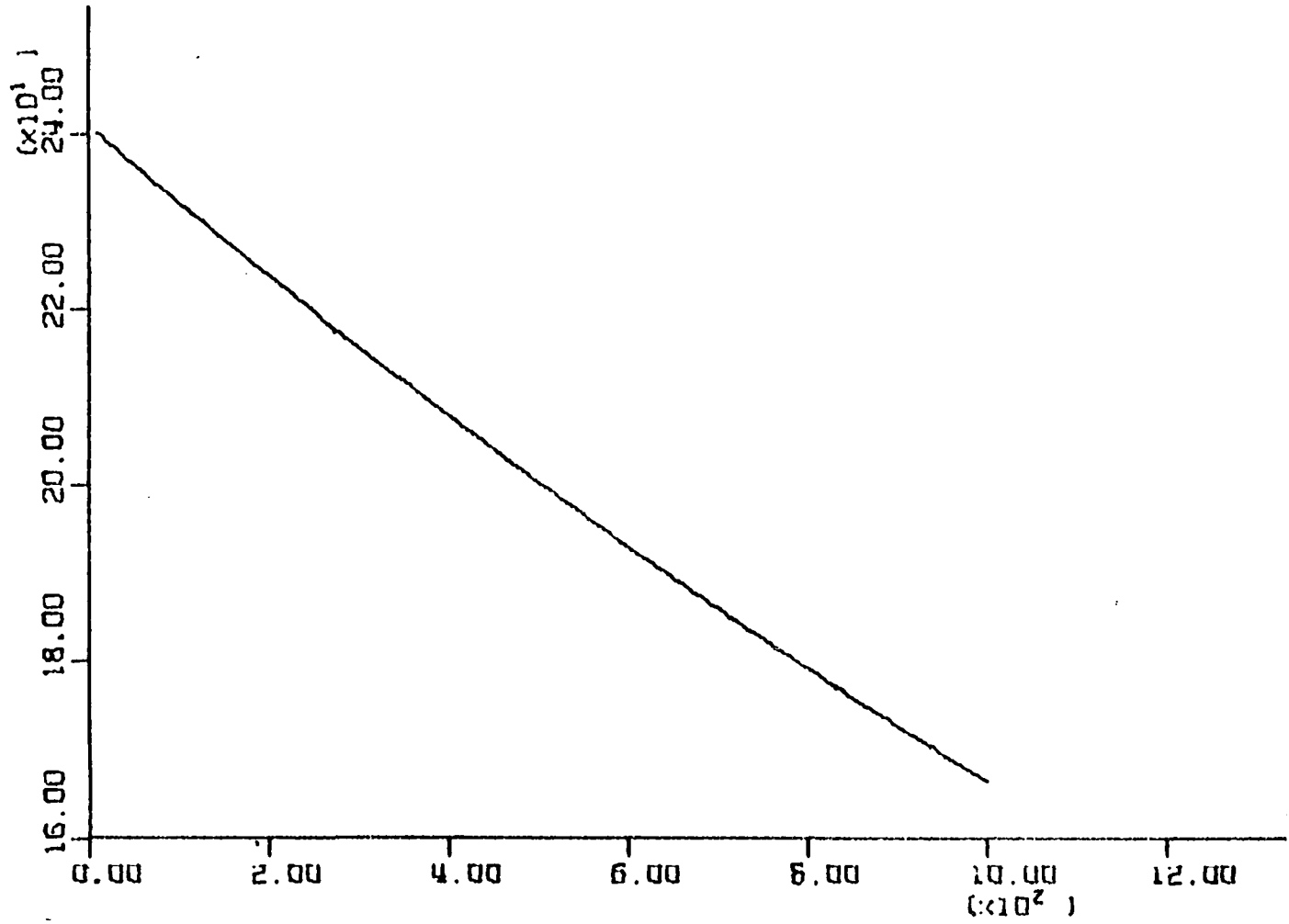


Fig. G.5. Total cost for two units



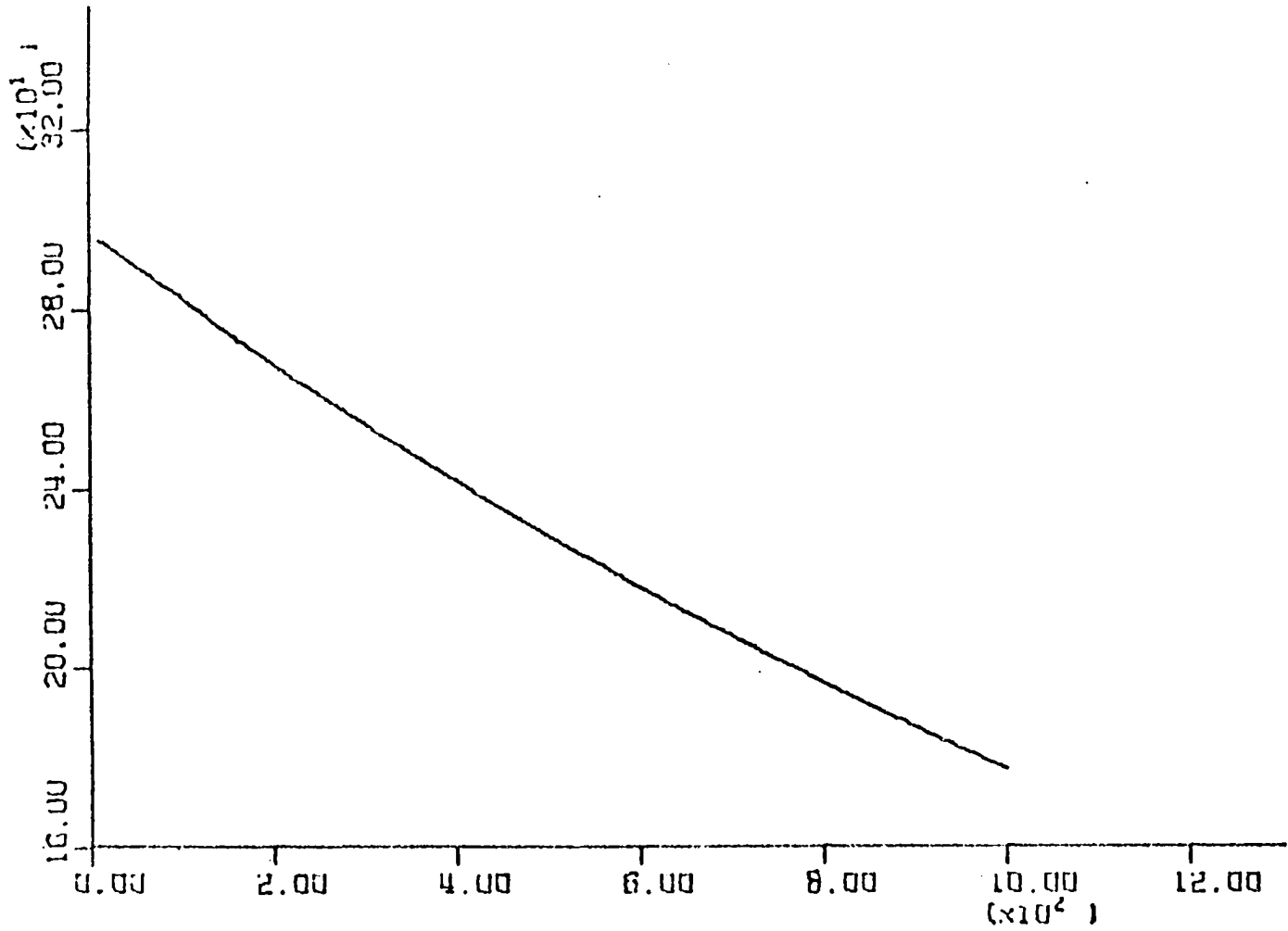


Fig. G.6. Total cost for one unit

The total costs in \$/Kw for one unit and two units of the same size are shown in Fig. G.5 and Fig. G.6 respectively.

#### Dynamic programming

For the year 1971, three different addition patterns are computed. A 400 MW unit is added to the original system and the costs computed. The capacity factor of that unit is 45% as calculated in Appendix F. The same cost computations are performed for a two units each of 200 MW capacity to be located in one location as the second choice and to be located in different places as the third choice. The costs are shown in Table G.4 considering the present worth costs based on year 1970 prices.

For the year 1972, the three different patterns are again studied under the assumption that the 400 MW unit added has been added the previous year. The costs are shown in Table G.5. Also, for the same year, the same three expansion plans are added to the system assuming the two 200 MW units added the previous year. The costs are shown in Table G.6.

A flow diagram is shown in Fig. G.7 which shows the nodes and the branches connecting these nodes. Node I represents the initial starting point and is called the source node. Nodes  $a_1$ ,  $b_1$ , and  $c_1$  represent the three different patterns chosen, i.e. the 400 MW, the 2-200 MW at one location and the 2-200 MW at two different locations respectively. The subscript 1 stands for the first stage or year 1971. The nodes  $a_2$ ,  $b_2$

and  $c_2$  are the same as  $a_1$ ,  $b_1$ , and  $c_1$  except that they correspond to the second stage or year 1972.

The same additions could be studied for any number of years. As an illustration of the method, these two stages will be sufficient to explain the use of the dynamic programming technique as discussed in Chapter VIII. The selection of the optimum project or pattern will be accomplished as follows:

For the first stage, we do not know what path we select that will optimize the costs so we must store the three patterns and their costs as shown in Fig. G.7.

For the second stage, if the costs of the branches between all nodes are added, we will have three values at each node (see Fig. G.8). We now select the minimum cost at each node and discard the other two costs. The optimality principle tells us that if it later turns out that the optimum policy will take us through state  $a_2$  for example, then we certainly must get there from state  $b_1$  as shown in Fig. G.9. That is why we discard the other two costs because the future optimum path will never pass through states that are not in the path of a candidate to an optimum solution. Doing this, we reduce the number of combinations from 27 for the third stage to just 9. The process will continue till the end of the period under study and we will have the only optimum path between the source and the sink nodes.

It should be mentioned that although dynamic programming

will select the optimum pattern from the available patterns, care should be given to the number of selections at every state and the number of nodes at every stage and finally the number of stages themselves.

The size and the number of units in every stage should be selected to satisfy the reliability constraint and at the same time decrease the overall costs.

The computer flow chart is shown on Fig. G.10 while the computer list is shown at the end of Appendix E. This computer list contains the LOLP, LOCP, the spinning reserve, the capacity factors of all the operating units, the production costs and the costs of the new additions.

If we examine the three costs shown with double asterisks in Fig. G.9, we find that the cost in state  $a_2$  exceeds that in state  $b_2$  by \$8,549,200, while the cost in state  $c_2$  exceeds the same cost in state  $b_2$  by \$18,379,000. These excesses in costs expressed as percentages of the cost in state  $b_2$  will be 2.96% and 6.36% respectively. This confirms that any small change in percent of the cost results in a considerable amount of millions of dollars which could be saved by proper selecting of the optimum pattern and the dynamic programming technique helps us in saving such money.

Table G.4. Costs for year 1971 (in k\$)

Item	One unit 400MW	2 x 200MW units (the same location)	2 x 200MW units (two locations)
Fuel costs	34,723.5700	33,497.4000	33,497.4000
Operation & maintenance costs	12,488.3000	12,788.1900	12,788.1900
New addition costs	95,359.9300	88,980.7200	105,585.8000
Fixed charges	11,919.9900	11,122.5900	13,198.225
Total cost of new addition	107,279.9200	100,103.3100	118,783.0250
Total P.W. cost (70)	154,491.7900	146,388.9000	165,668.6150
Total installed capacity	2960MW	2960MW	2960MW
Max. peak load demand	2675MW	2675MW	2675MW
Spinning reserve	640MW	490MW	490MW
Project code	A.1	BB.1	B.1

Cost of production and new additions.

Constants used:

1. Inflation factor 3.0%
2. Fixed charges 12.5%
3. Interest rate 7.0%
4. Present worth year 1970

Table G.5. Costs for year 1972 (in k\$)

Item	A.1 - 400	A.1 - 2 x 200 (1)	A.1 - 2 x 200 (2)
Fuel costs	35,499.2000	34,928.9100	34,928.9100
Operation & maintenance costs	13,848.6500	14,148.5500	14,148.5500
New addition costs	98,220.6800	91,650.1420	108,753.3800
Fixed charges	12,277.5200	11,456.2670	13,594.1200
Total cost of new addition	110,498.2000	103,106.4090	122,347.5000
Total P.W. cost (70)	149,777.1000	142,227.9000	160,606.9000
Total installed capacity	3360MW	3360MW	3360MW
Max. peak load demand	2884MW	2884MW	2884MW
Spinning reserve	780MW	700MW	700MW
Project code	A.1 - A.2	A.1 - BB.2	A.1 - B.2

Table G.6. Costs for year 1972 (in k\$)

Item	B - 400	B - 2 x 200 (1)	B - 2 x 200 (2)
Fuel costs	34,426.3800	33,846.5600	33,846.5600
Operation & maintenance costs	14,148.5500	14,448.4400	14,448.4400
New addition costs	98,220.6800	91,650.0600	108,753.3700
Fixed charges	12,277.5200	11,455.2400	13,594.1710
Total cost of new addition	110,498.2000	103,106.3000	122,347.5410
Total P.W. cost (70)	149,063.1000	141,901.6000	159,479.0001
Total installed capacity	3360MW	3360MW	3360MW
Max. peak load demand	2884MW	2884MW	2884MW
Spinning reserve	620MW	504MW	504MW
Project code	B.1 - A.2	B.1 - BB.2	B.1 - B.2

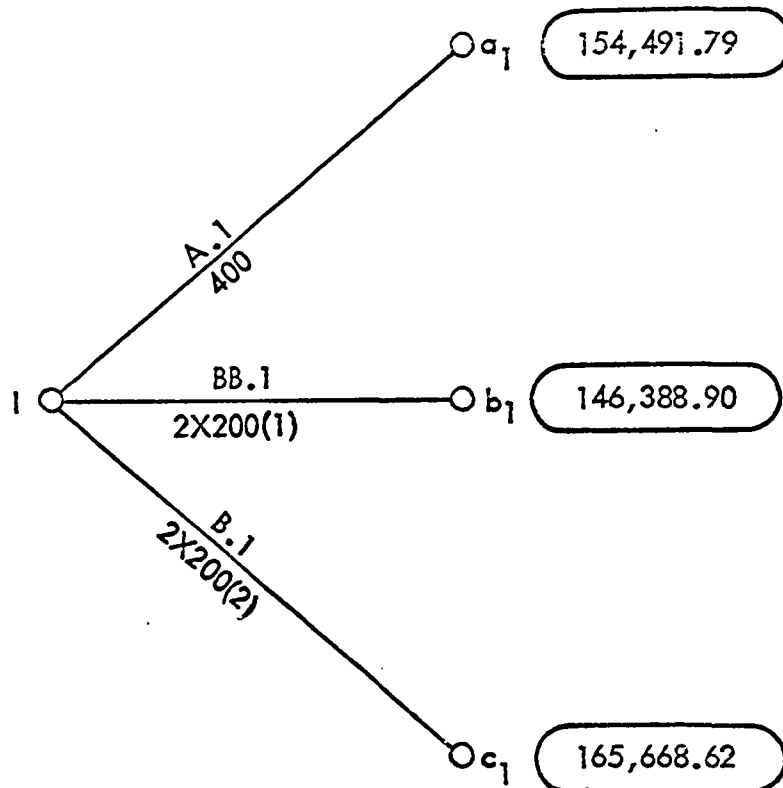


Fig. G.7. The first stage in dynamic programming to select the optimum pattern



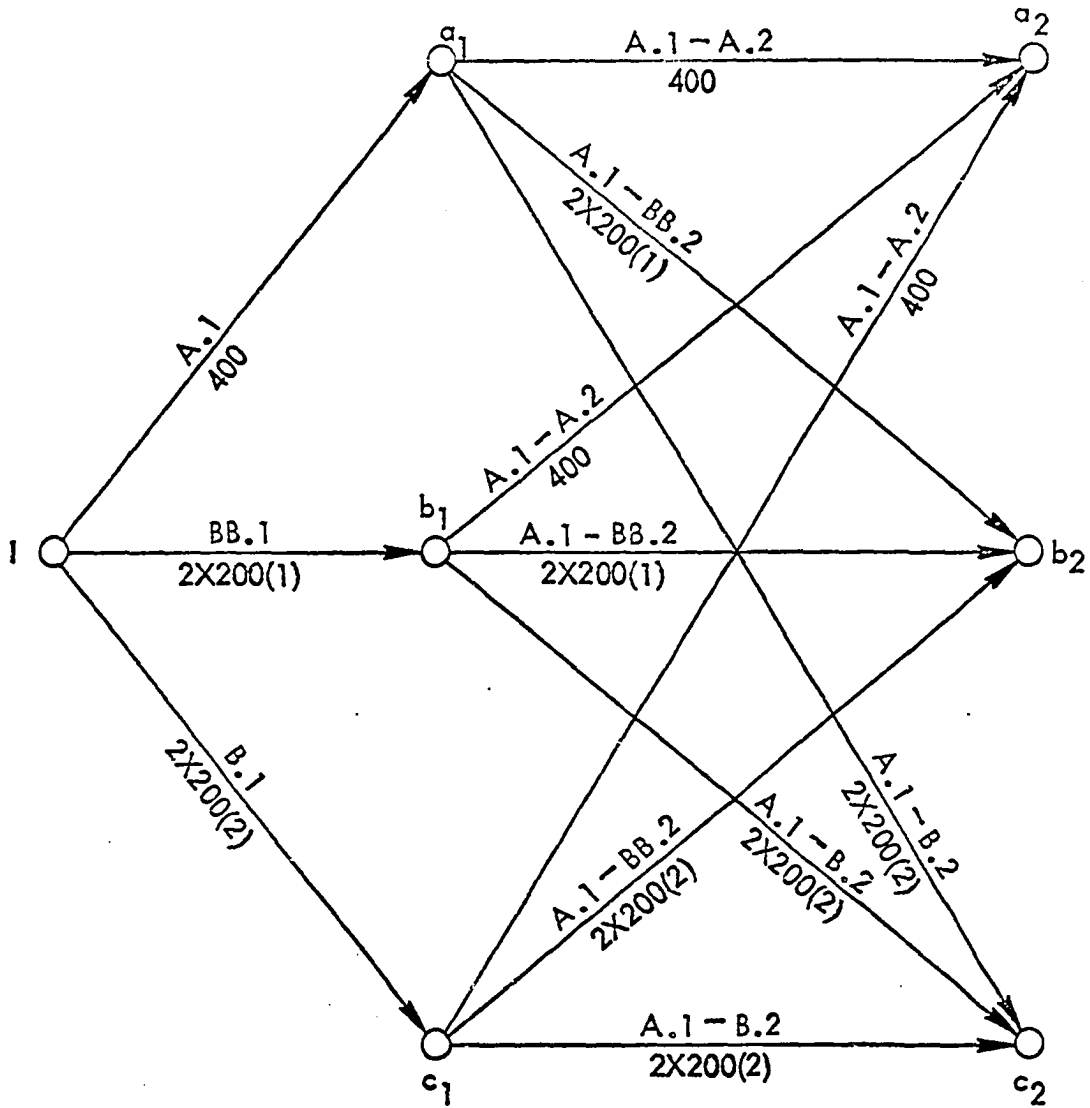


Fig. G.8. Flow diagram showing the nodes and the branches

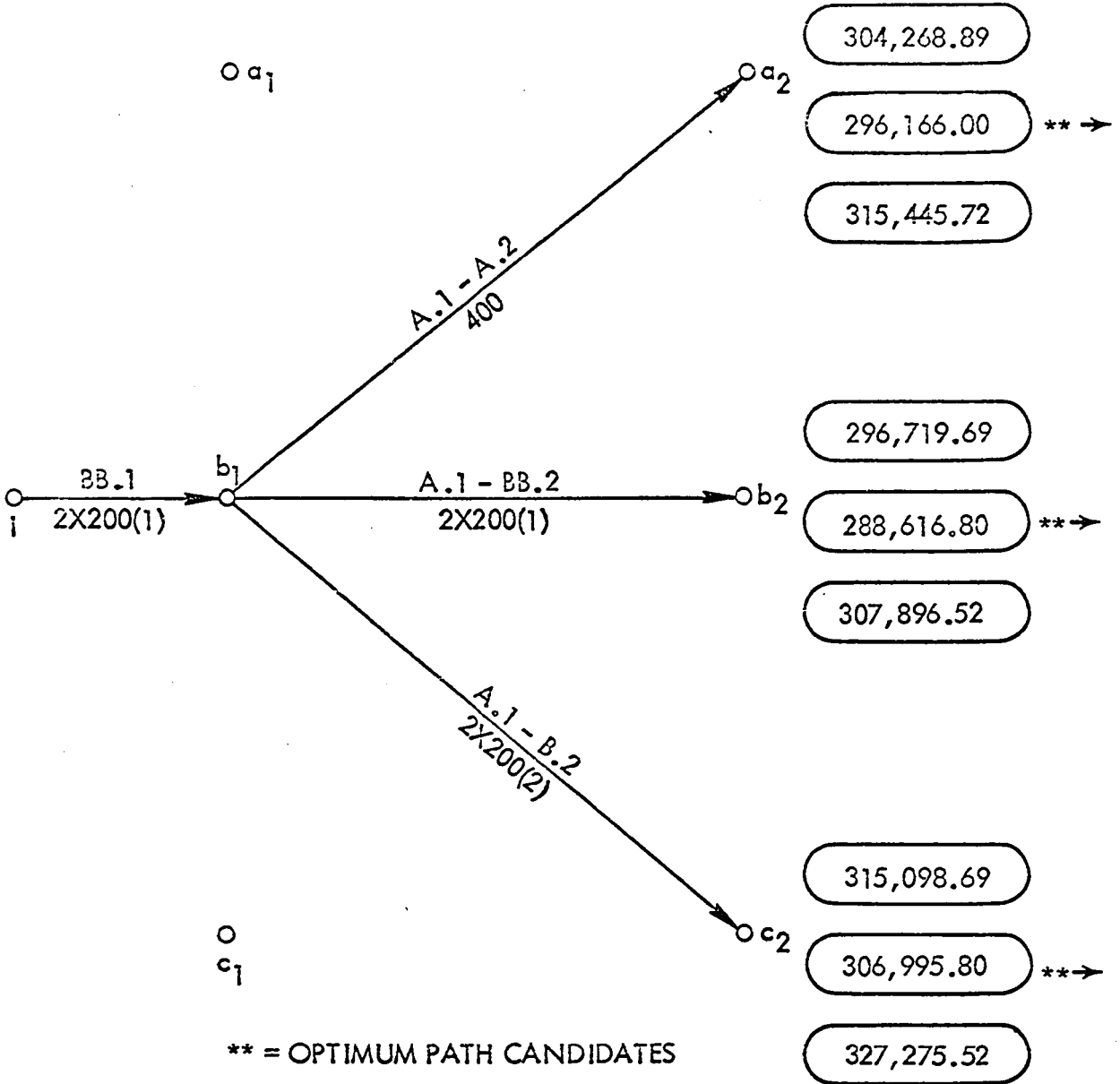


Fig. G.9. The second stage in dynamic programming to select the optimum pattern

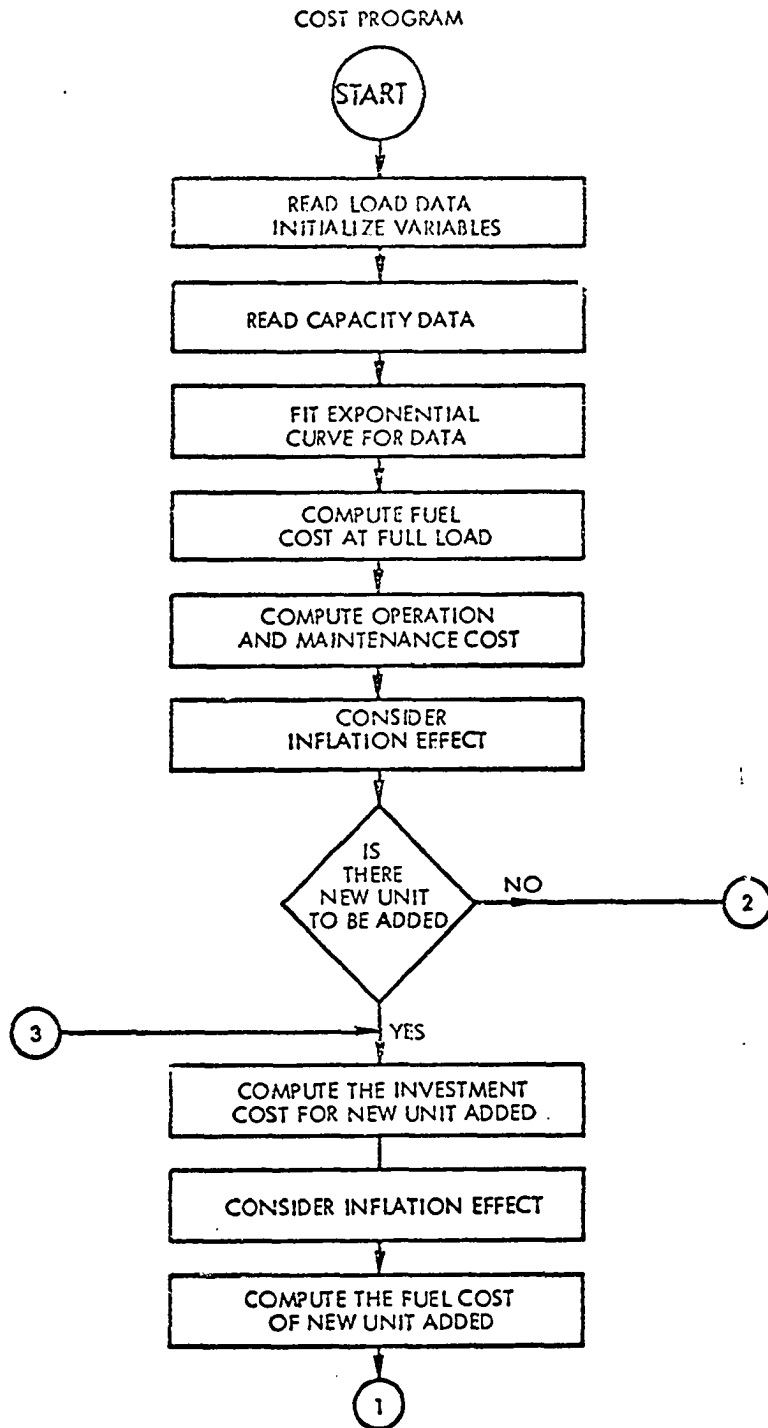


Fig. G.10. Cost program flow chart

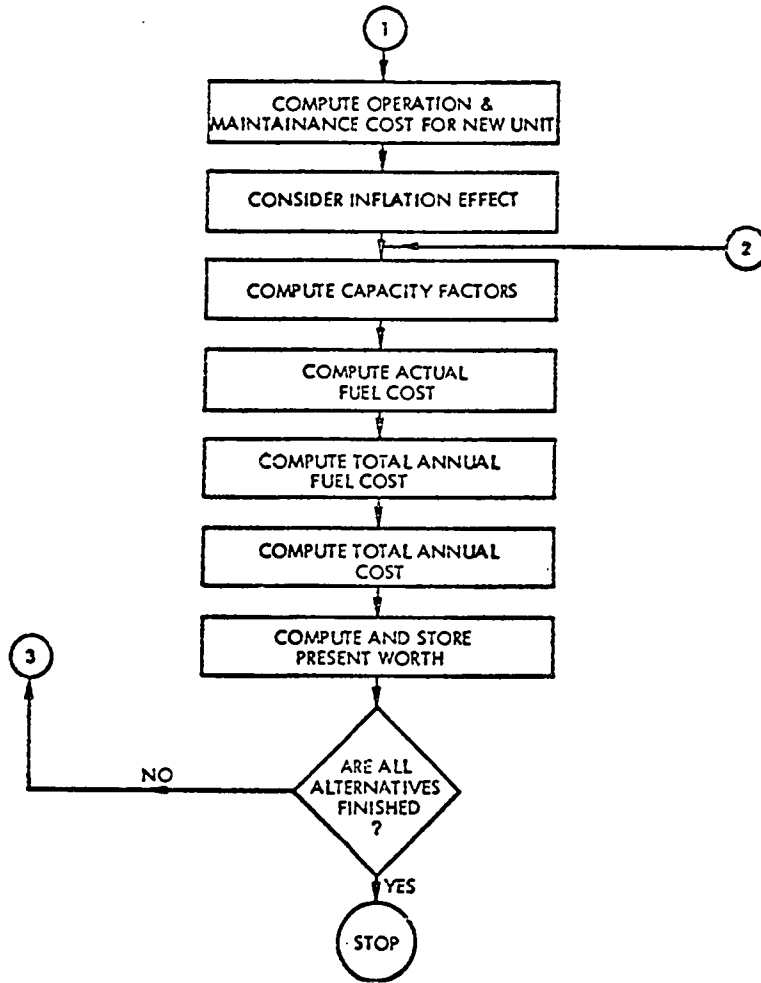


Fig. G.10. (Cont.)